Vocabulary:

2, 7, 12, 17, 22, 27, ...

Arithmetic Sequence - a pattern of numbers where the change is adding or subtracting the same number. We call this the common difference "d".

$d = 5$

$\boxed{\text{Closed/Explicit Formula}}$ - a formula for a sequence that is defined by the number of the term and goes directly to that term.

Recursive Formula - a sequence of numbers created by defining a term in the sequence and the pattern created by the sequence using previous terms.

Subscript - a number on a variable that identifies which variable it is. Example: x_2 means the second x.

$$\underset{a_1}{2}, \underset{a_2}{7}, \underset{a_3}{12}, \underset{a_4}{17}, \underset{a_5}{22}, \underset{a_6}{27}, \underset{a_7}{32}, \ldots$$

$a_3 = 12$ $(3, 12)$

$a_6 = 27$ $(6, 27)$

Arithmetic Sequences

F.BF.1a Determine an explicit expression and the recursive process (steps for calculation) from context.

F.BF.2 Write arithmetic and geometric sequences recursively and explicitly, use them to model situations, and translate between the two forms. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.

F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

What am I learning today?

How to recognize and write the formula for an arithmetic sequence

How will I show that I learned it?

Write a recursive and an explicit formula for an arithmetic sequence

Two ways to define sequences:

Closed/Explicit:

$a_n = a_1 + d(n-1)$

$a_n = a_1 + (n-1)d$

- can directly find nth term
- NOT required to list 1st term
- uses formula $\boxed{a_n = a_1 + d(n - 1)}$ for arithmetic sequences

Recursive:
- relates each term in the seq. to a previous term
- must ALWAYS state 1st term
- requires formula that relates the nth term to the (n-1)th term

Arithmetic Sequences

Sequences that are created by ADDING OR SUBTRACTING the same value. We call this value the COMMON DIFFERENCE. When graphed, it looks like a LINEAR FUNCTION with the difference related to the slope.

Example: 9, 7, 5, 3, 1, ...

Parts of a Sequence

terms → a_1 a_2 a_3 a_4 a_5

9, 7, 5, 3, 1, ...

$\underbrace{9, 7}_{-2}\underbrace{\,, 5}_{-2}\underbrace{\,, 3}_{-2}\underbrace{\,, 1}_{-2}$

Common difference $d = -2$

$\boxed{a_n = a_1 + d(n-1)}$

n^{th} term

1st term

common diff

stay

$(4, 3)$

$x \quad , \quad y$

$a_x = y$

$n = x$

Domain and Range of a Sequence:

Domain: The whole numbers of the terms in your sequence.
Range: The terms of the sequence.

Example: 9, 7, 5, 3, 1
Domain: {1, 2, 3, 4, 5} (5 terms in the sequence)
Range: {9, 7, 5, 3, 1} (same as the sequence itself)

Steps for finding the closed/explicit form:

Option 1
Use Formula

Formula: $\boxed{a_n = a_1 + d(n-1)}$

1) Plug in a_1 and d from your sequence.

2) Distribute your d and combine like terms.

$d = -2$

Example: 9, 7, 5, 3, 1, ...

$a_n = 9 + (-2)(n-1)$

$a_n = a_1 + d(n-1)$

$a_n = 9 - 2n + 2$

$\boxed{a_n = -2n + 11}$

$a_{200} = -2(200) + 11$

$= -389$

~~Option 2~~
~~Treat like a Linear Function~~

~~1) Find the difference "d" between each term.~~

~~2) Subtract this "d" from the first term to find a_0.~~

~~3) Plug into form $a_n = dn + a_0$ (like $y = mx + b$)~~

Example: 9, 7, 5, 3, 1, ...

$y = mx + b$

$a_n = -2n + 11$

Closed/Explicit definition: $a_n = a_1 + d(n-1)$

Ex 1: 5, 10, 15, 20, 25 ...

$\boxed{a_n = 5n}$

$a_1 = 5$

$d = 5$

$a_n = 5 + 5(n-1)$

$a_n = 5 + 5n - 5$

$a_n = 5n$

D: $\{1, 2, 3, 4, 5, \ldots\}$

R: $\{5, 10, 15, 20, 25, \ldots\}$

Ex 2: 2, -4, -10, -16, -22 ...

$\boxed{a_n = -6n + 8}$

$a_1 = 2$

$d = -6$

$a_n = 2 + (-6)(n-1)$

$a_n = 2 - 6n + 6$

$a_n = -6n + 8$

D: $\{1, 2, 3, 4, 5, \ldots\}$

R: $\{2, -4, -10, -16, -22, \ldots\}$

Closed/Explicit definition:

Ex 3: 2, 8, 14, 20, 26 ...

$a_1 = 2$

$d = 6$

$$\boxed{a_n = 6n - 4}$$

D: {1, 2, 3, 4, 5, ...}

R: {2, 8, 14, 20, 26, ...}

Ex 4: 30, 25, 20, 15, 10 ...

$a_1 = 30$ $d = -5$

$$\boxed{a_n = -5n + 35}$$

D: {1, 2, 3, 4, 5, ...}

R: {30, 25, 20, 15, 10, ...}

Ex 5: Given the explicit definition, find the first five terms of the sequence. Then, find the 20th term.

$a_n = 4n - 1$ $d=4$ $a_{20} = 4(20)-1$

$= 79$

$a_1 = 4(1)-1 = 3$

$a_2 = 4(2)-1 = 7$

$a_3 = 11$

$a_4 = 15$

$a_5 = 19$

Ex 6: Given the explicit definition, find the first five terms of the sequence. Then find the 20th term.

$a_n = -3n + 8$ $d = -3$

$5, 2, -1, -4, -7$

$a_1 = -3(1) + 8 = 5$

$a_2 = 2$

$a_3 = -1$

$a_4 = -4$

$a_5 = -7$

$a_{20} = -3(20) + 8$

$= -52$

Ex. 7 $a_3 = 6$ $d = -4$

What is the 20th term of the sequence?

$\underline{14}, \underline{10}, \underline{6}, \underline{2}$

$a_1 \quad a_2 \quad a_3 \quad a_4$

$a_1 = 14 \quad d = -4$

$a_n = 14 + (-4)(n-1)$

$a_n = -4n + 14 + 4$

$a_n = -4n + 18$

$a_{20} = -62$

$a_n = a_1 + d(n-1)$

must match

$a_n = a_3 + d(n-3)$

must match

$a_{20} = 6 + (-4)(20-3)$

n a_3 n

$a_{20} = 6 - 68 = -62$

$$\boxed{a_n = a_1 + d(n-1)}$$

Ex. 8 $a_4 = 12$ $a_7 = 27$

$\underset{a_4}{\underline{12}}\ \underset{a_5}{\underline{17}}\ \underset{a_6}{\underline{22}}\ \underset{a_7}{\underline{27}}$

$d = 5$

What is the 200th term of the sequence?

$(4, 12)$ $(7, 27)$

$$d = \text{slope} = \frac{27-12}{7-4} = \frac{15}{3} = 5$$

$$a_n = a_4 + d(n-4)$$

$$a_n = 12 + 5(n-4)$$

$$a_{200} = 12 + 5(200-4)$$

$$= 992$$

HW: #1-18