Factoring and Solving Polynomials using the Factor Theorem

The remainder and factor theorems can be used to find the zeros of a polynomial when factoring is not always possible.

Essential Question: **When dividing a polynomial with another polynomial, what is the remainder of zero telling you?**

MCC9-12.A.APR.2 Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x - a is p(a), so p(a) = 0 if and only if (x - a) is a factor of p(x).

MCC9-12.A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Finding the roots of polynomials

Solutions

To find all the roots/ zeros of a polynomial, several theorems have to be used.

A. Remainder Theorem

*If a poly. P(x) is divided by (x - a), then the remainder is P(a)

(x-1)

P(1)

ex. If $P(x) = x^3 - 7x^2 - 4$ is divided by (x + 1), then P(-1) is the remainder.

$$\begin{array}{r|rrrr} -1 & 1 & -7 & 0 & -4 \\ & \downarrow & -1 & 8 & -8 \\ \hline & 1 & -8 & 8 & -12 \end{array}$$

R -12

$P(-1) = (-1)^3 - 7(-1)^2 - 4 = -12$

B. Factor Theorem

*if the remainder is zero,

then (x - a) is a FACTOR of P(x)

ex. Is (x - 5) a factor of

$f(x) = x^3 - 4x^2 - 7x + 10$?

$f(5) = (5)^3 - 4(5)^2 - 7(5) + 10$ ff below

$= 0$

$$\begin{array}{c|cccc} 5 & 1 & -4 & -7 & 10 \\ & \downarrow & 5 & 5 & -10 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

yes (x-5) is a factor

FF: f(x) = (x - 5)(x + 2)(x - 1)

SUMMARY

$f(a) = b$

represents **substituting** into the function a and getting the result b

is one of the ordered pairs of the given polynomial such that (a, b) exists on the graph (x, y)

$f(a) = b$ ← remainder

represents **dividing** the function by $(x - a)$ and getting a remainder of b. $(x-2)$ $f(2) = b$

$f(\#) = 0 \leftarrow$ remainder

__\#__ represents one of the **x-intercepts** of the graph

__\#__ represents one of the **real solutions / zeros / roots** of given polynomial such that ...

$$f(x) \div (x - \#) = 0$$

which means it is a **factor** of the polynomial

Given f(x) what are the following telling you?

a. $f(3) = 9$ remainder

$(3, 9)$ ordered pair on graph of $f(x)$

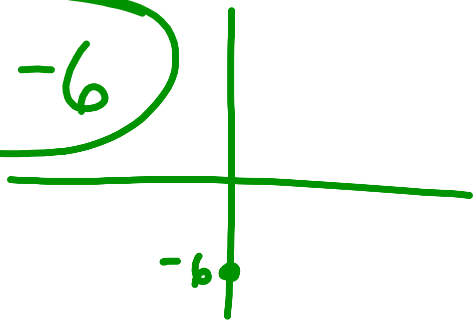
$f(x) \div (x-3) = 9$ remainder

b. $f(0) = -6$

$(0, -6)$ y-int of $f(x)$

$f(x) \div (x-0)$ $R = -6$

$f(x) \div x$



c. $f(-2) = 0$

$(-2, 0)$ ordered pair ∴ x-int of graph

$f(x) \div (x+2)$ $R = 0$

$(x+2)$ is a factor of $f(x)$

$x = -2$ is a root, solution, zero of $f(x)$

FACTORING

The remainder and factor theorems can be used to find the zeros of a polynomial when factoring from the beginning is not possible.

Given the polynomial and one zero of the function, factor completely.

#1 $f(x) = 2x^3 + 11x^2 + 18x + 9;\ -3$

$(x+3)$

This is a factor

$$\begin{array}{r|rrrr} -3 & 2 & 11 & 18 & 9 \\ & \downarrow & -6 & -15 & -9 \\ \hline & 2 & 5 & 3 & 0 \end{array}$$

$2x^2 + 5x + 3$

Factor completely

+6	+5
+3 · +2	3+2

$(2x^2 + 2x)(+3x + 3)$

$2x(x+1) + 3(x+1)$

given

FF: $\boxed{(x+1)(2x+3)(x+3)}$

#2

$(3x+2)$ factor

$$g(x) = 3x^5 + 2x^4 - 18x^3 - 12x^2 + 15x + 10$$

where $g(-2/3) = 0$

is a zero, root, x-int

$$\begin{array}{r|rrrrrr} -\frac{2}{3} & 3 & 2 & -18 & -12 & +15 & 10 \\ & \downarrow & -2 & 0 & 12 & 0 & -10 \\ \hline & 3x^4 & 0x^3 & -18x^2 & 0x & 15 & 0 \end{array}$$

R

$$\boxed{3x^4 - 18x^2 + 15}$$

completely factor

$$3(x^4 - 6x^2 + 5)$$

+5	-6
$-5 \cdot -1$	$-5 + -1$

$$(x^4 - 5x^2)(-x^2 + 5)$$

$$x^2(x^2 - 5) - 1(x^2 - 5)$$

$$3(x^2 - 5)(x^2 - 1)$$

D.O.T.S.

F.F.

$$3(x^2 - 5)(x+1)(x-1)(3x+2)$$

given

3

Start w/ smallest

Given the factors $(x - 1)$ and $(x - 3)$,

$f(x) = x^4 - 10x^3 + 35x^2 - 50x + 24$

$$\begin{array}{r|rrrrr} 1 & 1 & -10 & 35 & -50 & 24 \\ & \downarrow & 1 & -9 & 26 & -24 \\ \hline & 1 & -9 & 26 & -24 & 0 \end{array}$$

$\boxed{x^3 - 9x^2 + 26x - 24}$

$$\begin{array}{r|rrrr} 3 & 1 & -9 & 26 & -24 \\ & \downarrow & 3 & -18 & 24 \\ \hline & 1 & -6 & 8 & 0 \end{array}$$

$x^2 - 6x + 8$

$(x-4)(x-2)$

+8	-6
$-4 \cdot -2$	$-4 + -2$

F.F. = $(x-4)(x-2)(x-1)(x-3)$