

Operations with Functions

EQ: Can you create new functions by using basic operations?

Standards:

Build a function that models a relationship between two quantities

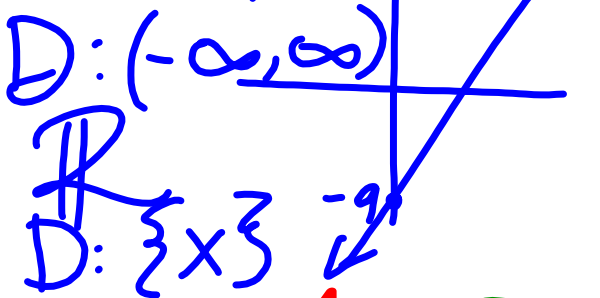
MCC9-12.F.BF.1b Combine standard function types using arithmetic operations.

MCC9-12.F.BF.1c Compose functions.

Recall: Given the following what types of functions, what is the domain?

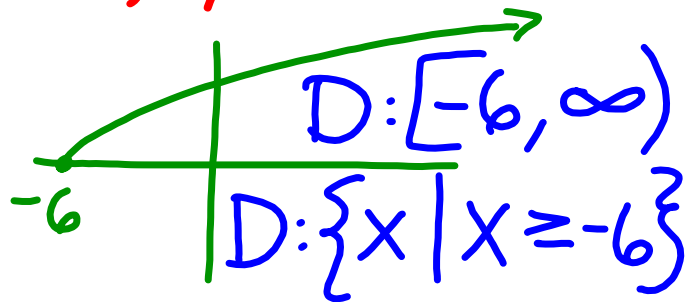
1) $y = 3x - 9$

Linear



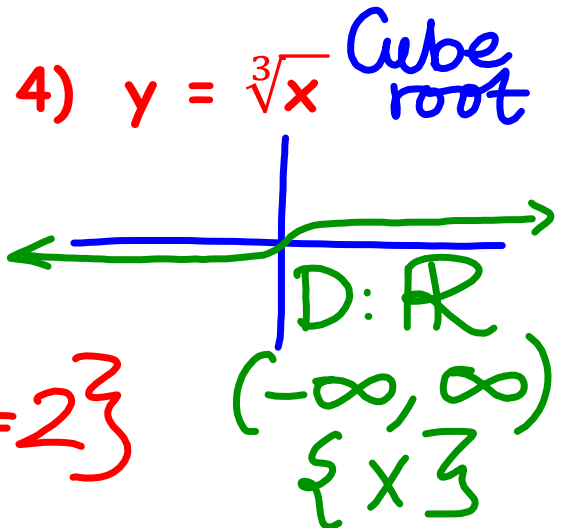
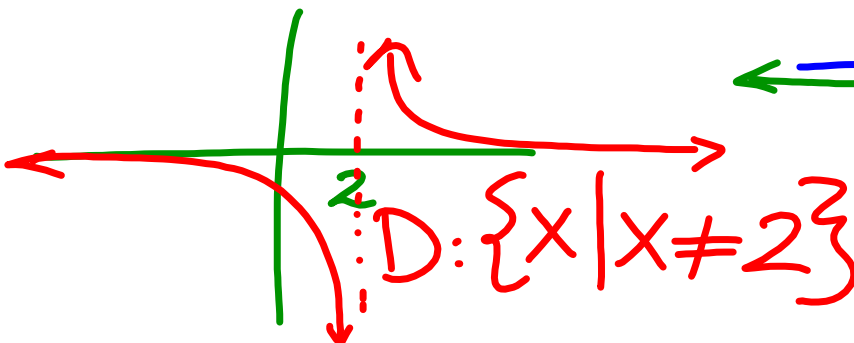
Square Root

2) $y = \sqrt{x + 6}$



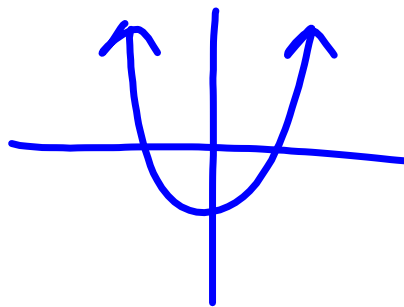
3) $y = \frac{4}{x - 2}$

Rational



5) $y = x^2 + 3x - 9$ Quadratic

Domain: \mathbb{R}
 $(-\infty, \infty)$
 $\{x\}$



Given 2 functions $f(x)$ and $g(x)$, a new function, $h(x)$ can be defined performing any of the 4 basic operations.



Notation for Function Operations	
Operation	Notation
Addition	$(f + g)(x) = f(x) + g(x)$
Subtraction	$(f - g)(x) = f(x) - g(x)$
Multiplication	$(fg)(x) = f(x) \cdot g(x)$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$

****Must give exclusions for division problems!!**

Given $g(x) = 3x^2 - 2x - 1$ and $f(x) = 3x + 4$, find each function. State the domain for each.

$$\begin{aligned} 1. (f - g)(x) &= f(x) - g(x) \\ &= (3x + 4) - (3x^2 - 2x - 1) \\ &= 3x + 4 - 3x^2 + 2x + 1 \\ &= -3x^2 + 5x + 5 \end{aligned}$$

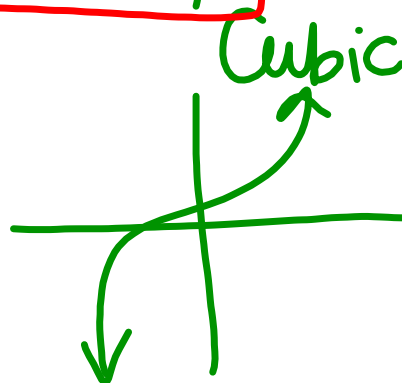
Domain: \mathbb{R} Quadratic
 $(-\infty, \infty)$ or $\{x\}$

Given $g(x) = 3x^2 - 2x - 1$ and $f(x) = 3x + 4$, find each function. State the domain for each.

$$\begin{aligned} 2. (gf)x &= g(x) \cdot f(x) \\ &= (3x^2 - 2x - 1)(3x + 4) \end{aligned}$$

$$9x^3 + 6x^2 - 11x - 4$$

$$\begin{aligned} D: &\mathbb{R} \\ &(-\infty, \infty) \\ &\{x\} \end{aligned}$$



Given $g(x) = 3x^2 - 2x - 1$ and $f(x) = 3x + 4$,
find each function. State the domain for each.

$$3. \left(\frac{g}{f} \right) x = \frac{g(x)}{f(x)} = \frac{3x^2 - 2x - 1}{3x + 4}$$

factor \downarrow $(x-1)(3x+1)$

$$\begin{array}{r|l} -2 & -2 \\ \hline -3 \cdot 1 & -3+1 \end{array}$$

factor top: bottom
See if something
cancels

$$= \frac{(x-1)(3x+1)}{3x+4}$$

$$\begin{array}{r} 3x+4=0 \\ \hline -4 \quad -4 \\ \hline 3x = -4 \\ \frac{3x}{3} = \frac{-4}{3} \\ x = -\frac{4}{3} \end{array}$$

$$D: \left\{ x \mid x \neq -\frac{4}{3} \right\}$$

Given $g(x) = 3\sqrt{x} + 5$ and $f(x) = 9\sqrt{x} - 7$, find each function. State the domain for each

$$\begin{aligned} 1. (g + f)x &= g(x) + f(x) \\ &= (3\sqrt{x} + 5) + (9\sqrt{x} - 7) \\ &= 12\sqrt{x} - 2 \\ D: \{x \mid x \geq 0\} \end{aligned}$$

Given $g(x) = 3\sqrt{x} + 5$ and $f(x) = 9\sqrt{x} - 7$,
find each function. State the domain for
each

$$\begin{aligned} 2. (g \cdot g)(x) &= g(x) \cdot g(x) \\ &= (3\sqrt{x} + 5)(3\sqrt{x} + 5) \\ &= 9x + 30\sqrt{x} + 25 \\ &D: \{x \mid x \geq 0\} \end{aligned}$$

Another operation is the composite of functions

$$(f \circ g)(x) = f(g(x))$$



Composition of Functions

The composition of functions f and g is notated

$$(f \circ g)(x) = f(g(x)).$$

The domain of $(f \circ g)(x)$ is all values of x in the domain of g such that $g(x)$ is in the domain of f .

Reading Math

The composition $(f \circ g)(x)$ or $f(g(x))$ is read "f of g of x."



→ **What is $(g \circ f)$?**

$$(f \circ g)(x) = f(g(x))$$

Domain is the combo for the two domains.

$$f(x) \Rightarrow D: \{x / x \neq 0\} \text{ (rational)}$$

$$g(x) \Rightarrow D: x \geq 0 \text{ (square root)}$$

$$f(g(x)) \Rightarrow D: \{x / x > 0\}$$

$$f(x) = x^2 + 2$$

$$f(2) = (2)^2 + 2 =$$

$$f(x) \quad g(x)$$

Evaluating composites:

Find the value of the interior function.

Plug new value into the outer function.

OR set up new function and plug in.

Given $f(x) = 4x + 13$ and $g(x) = 3x^2 - x$,
evaluate the following. $g(f(3)) = 1850$

1. $g(\underline{f(3)})$

$$f(3) = 4(3) + 13 \\ = 25$$

$$g(25) = 3(25)^2 - 25 = 1850$$

Evaluating basic ops:

Set up the new function .

Plug given value in as x and evaluate.

Given $f(x) = 4x + 13$ and $g(x) = 3x^2 - x$,
evaluate the following.

$$\begin{aligned} 2. \quad gf(3) & \quad (3x^2 - x)(4x + 13) \\ & \quad 12x^3 + 39x^2 - 4x^2 - 13x \\ & \quad 12x^3 + 35x^2 - 13x \\ & \quad 12(3)^3 + 35(3)^2 - 13(3) \\ & \quad = 600 \end{aligned}$$

Given $f(x) = \sqrt{4x + 13}$ and $g(x) = 3x^2 - x$, evaluate the following.

3. $f(\cancel{g(-2)})$

$$g(-2) = 3(-2)^2 - (-2) = 14$$

$$f(14) = \sqrt{4(14) + 13} = \sqrt{69}$$

$$f(g(-2)) = \sqrt{69}$$

Using tables the evaluate:

x	1	2	3	4
f(x)	-3	-1	1	3

x	-1	0	1	2
g(x)	-4	1	4	5

1. $(f \circ f)(4) = 1$

~~$f(f(4))$~~
 $f(3) = 1$

2. $f(g(0)) = -3$
 $f(1) = -3$

3. $(g \circ f)(3) = 4$

$g(f(3))$
 $g(1) =$

4. $g(f(2)) = -4$

$g(-1) = -4$

Write the composite function & state its domain.

$$f(x) = x^2 + 3x - 8$$

$$g(x) = 2x - 3$$

1. $g(f(x))$

$$2x - 3$$

$$g(x^2 + 3x - 8) = 2(x^2 + 3x - 8) - 3$$

$$= 2x^2 + 6x - 16 - 3$$

$$g(f(x)) = 2x^2 + 6x - 19$$

$$D: \mathbb{R} \text{ or } (-\infty, \infty)$$

Write the composite function & state its domain.

$$f(x) = x^2 + 3x - 8$$

$$g(x) = \underline{2x} - 3$$

$$2. (g \circ g)x = g(\underline{g(x)})$$
$$g(\underline{2x-3}) = 2(\underline{2x-3}) - 3$$
$$= 4x - 6 - 3$$

$$D: \mathbb{R}$$

$$\boxed{g(g(x)) = 4x - 9}$$

Write the composite function & state its domain.

$$f(x) = \underline{x^2} + 3\underline{x} - 8$$

$$g(x) = \underline{2x - 3}$$

$$3. (f \circ g)(x) = f(\underline{g(x)})$$
$$f(2x-3)$$

$$f(2x-3) = \underline{(2x-3)^2} + 3(2x-3) - 8$$
$$= 4x^2 - 12x + 9 + 6x - 9 - 8$$

$$(f \circ g)(x) = 4x^2 - 6x - 8$$

$$D: \mathbb{R}$$

Write the composite and its domain.

Given: $f(x) = 3\sqrt{x-5}$ $g(x) = x^2 + 8$

$$g(f(x)) = g(3\sqrt{x-5}) =$$

$$g(3\sqrt{x-5}) = (3\sqrt{x-5})^2 + 8$$

$$= 9(x-5)$$

$$= 9x - 45 + 8$$

$$g(f(x)) = 9x - 37$$

D:
 \mathbb{R}

Write the composite and its domain.

Given: $f(x) = 4x^2 + 3$ $D: \mathbb{R}$ $g(x) = \frac{2x}{x+5}$ $D: \{x \mid x \neq -5\}$

$$\begin{aligned} (g \circ f)(x) &= g(f(x)) = g(4x^2 + 3) \\ &= \frac{2(4x^2 + 3)}{(4x^2 + 3) + 5} = \frac{8x^2 + 6}{4x^2 + 8} \quad \text{factor to reduce to} \\ &= \frac{\cancel{2}(4x^2 + 3)}{\cancel{2}(2x^2 + 4)} = \frac{4x^2 + 3}{2x^2 + 4} \end{aligned}$$

$$g(f(x)) = \frac{4x^2 + 3}{2x^2 + 4} \quad D: \{x \mid x \neq -5\}$$

must combine Domain from $g(x) \cap f(x)$

Write the composite and its domain.

Given: $f(x) = 3x^3 + 1$ $g(x) = \left(\frac{x-1}{3}\right)^{\frac{1}{3}}$ $D: \mathbb{R}$
 $D: \mathbb{R}$ Cuberoot

$$(g \circ f)x = g(f(x)) = g(3x^3 + 1)$$

$$= \left(\frac{3x^3 + 1 - 1}{3}\right)^{\frac{1}{3}} = \left(\frac{3x^3}{3}\right)^{\frac{1}{3}} = (x^3)^{\frac{1}{3}} = x$$

$$(g \circ f)x = x \quad D: \mathbb{R}$$

Write the composite and its domain.

Given: $f(x) = 3x^3 + 1$ $g(x) = \left(\frac{x-1}{3}\right)^{\frac{1}{3}}$

$$\begin{aligned}(f \circ g)x &= f(g(x)) = f\left(\left(\frac{x-1}{3}\right)^{\frac{1}{3}}\right) \\ &= 3\left(\left(\frac{x-1}{3}\right)^{\frac{1}{3}}\right)^3 + 1 = \cancel{3}\left(\frac{x-1}{\cancel{3}}\right) + 1 \\ &= x-1 + 1 = x\end{aligned}$$

$$(f \circ g)x = x \quad D: \mathbb{R}$$

HW: p. 438 - 440

2 - 13 all

16 - 26 evens,

30 - 32, 39 - 42, 45 - 47