Way back in the 17th century. . . .

Isaac Newton

Gottfried Wilhelm Leibniz

1) Tangent Line Problem
   Differential Calculus

2) Area Under Curves
   Integral Calculus

Limits

Intro to Derivatives

$(2,3)\ (-4,7)$

Can you solve for the slope between two points?!?!

How?

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{-4-2} = \frac{4}{-6} = -\frac{2}{3}$$

$y =$

$f(x) =$

Now in function notation...

$$slope = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

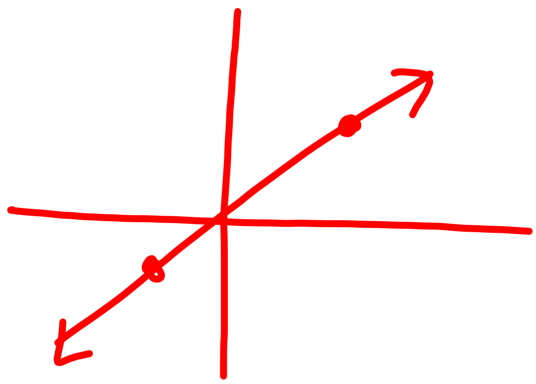
2.1 The Derivative and the tangent line problem

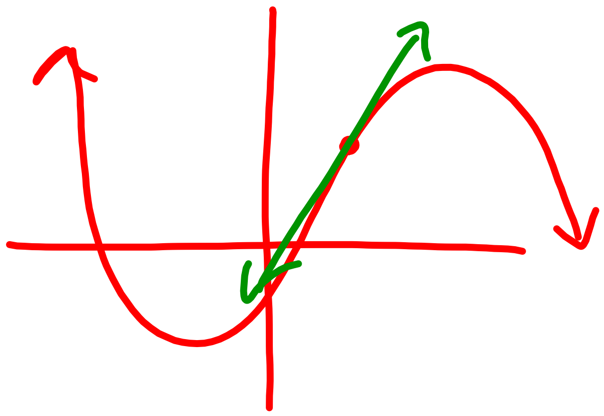
A derivative is used to find the slope of a tangent line to a curve. In other words, it finds the *instantaneous slope*.

We notate the derivative by $f'(x)$, read "f prime of x."

$f'(x)$ ← the derivative

$f(x) =$





Finding a derivative is called **differentiation**.

The derivative of f(x) can be written different ways:

$$f'(x) = y' = y'(x) = \cancel{\frac{df}{dx}} = \frac{dy}{dx} = \frac{d}{dx}f(x)$$

$y = 2x + 3$

y'

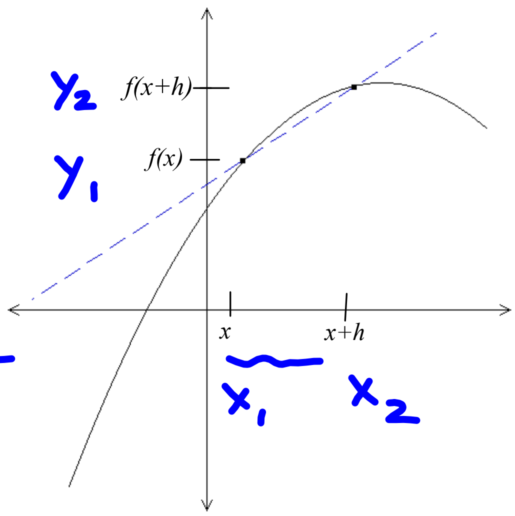
Leibniz Notation

* *dy* and *dx* are called differentials.
It doesn't mean *dy* divided by *dx*.

What is the slope of the line that goes through these two points? What happens as the points get closer to each other?

$$m = \frac{f(x+h) - f(x)}{\cancel{x} + h - \cancel{x}}$$

$$m = \frac{f(x+h) - f(x)}{h}$$

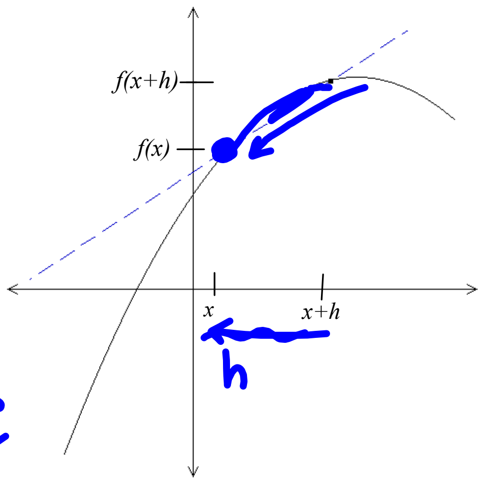


Slope of the Tangent Line = Derivative

$$\boxed{m = \lim_{h\to 0} \frac{f(\cancel{x}+h) - f(\cancel{x})}{h}}$$

This limit is called the "difference quotient."

instantaneous slope



The Definition of the Derivative

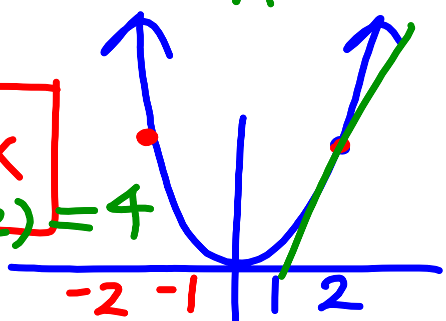
$$f'(x) = m = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

or

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example 1: $f'(x) = y' = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

Find the slope on $y = x^2$ at $x = 2$.



$f(x) = x^2$

$\boxed{f'(x)=2x}$

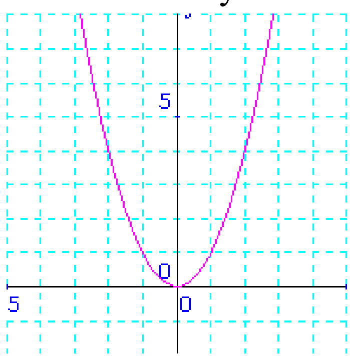
$f'(2)=2(2)=4$

$$\lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}$$

$$\lim_{h\to 0} \frac{2xh+h^2}{h} = \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x+0 = \boxed{2x}$$

The slope is: 4

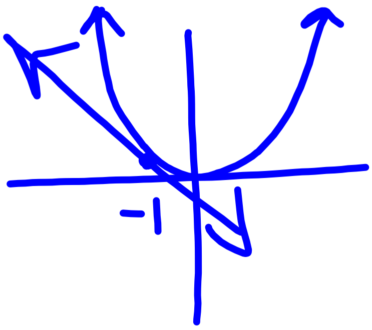
Now look at the graph of $y = x^2$ and check your answer. Does this look like the slope of the tangent line at x = 2?



$g(x) = 4x + 2$ $f(x) = x^2$

$g(f(x))$

Example 2: $y' = 2x$



$f(x)$

Find the slope on $y = x^2$ at $x = -1$.

$\lim_{h \to 0} f(x) = 2x$

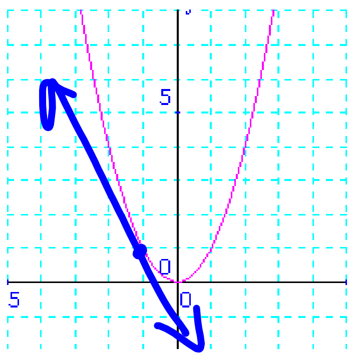
$m = 2x$ @ $x = -1$

$\boxed{f'(x) = 2x}$

$-2 = m$

The slope is: -2

Now look at the graph of $y = x^2$ and check your answer. Does this look like the slope of the tangent line at $x = -1$?

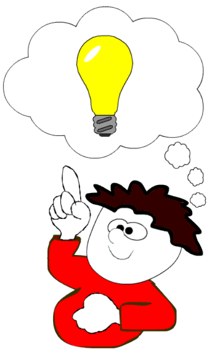


Example 3: $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Find the slope on y = 2x - 3 at x = 3.

$$\lim_{h \to 0} \frac{2(x+h) - 3 - (2x-3)}{h}$$

$$\lim_{h \to 0} \frac{\cancel{2x} + 2h - \cancel{3} - \cancel{2x} + \cancel{3}}{h} = \frac{2\cancel{h}}{\cancel{h}} = 2$$



How could we have done this in under 2 seconds flat?!?!

Example 4:

Write the equation of a line tangent to $\boxed{y = -2x^2}$ at x = 1

$y = -2x^2$

$y' = -4x$

What two things do we need to write the equation of a line?

1. Slope ✓ -4
2. Point $(1,-2)$

$\underbrace{y - y_1 = m(x - x_1)}_{\text{point-slope equation}}$

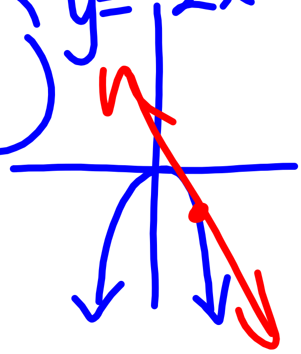
$x^2+2xh+h^2$

$$\lim_{h\to 0} \frac{-2(x+h)^2 - (-2x^2)}{h} =$$

$$\lim_{h\to 0} \frac{-\cancel{2x^2} - 4xh - 2h^2 + \cancel{2x^2}}{h} = \frac{\cancel{h}(-4x-2h)}{\cancel{h}}$$

$$\lim_{h\to 0} -4x - 2(0) = -4x$$

$y = -2x^2$



$f'(x) = f'(1) = -4(1)$

$= -4$

$(1, -2)$
$x_1 \quad y_1$

$m = -4$

$y - y_1 = m(x - x_1)$

$y - (-2) = -4(x - 1)$

$y + 2 = -4x + 4$
$\quad -2 \qquad\qquad -2$

$\boxed{y = -4x + 2}$

tangent line

$\boxed{f'(x)=6x-5}$

Use the definition of a derivative:

Find $f\,'(x)$ if $f(x) = 3x^2 - 5x^{1-1}$

$$\lim_{h\to 0} \frac{\overbrace{3(x+h)^2-5(x+h)}^{f(x+h)} - \overbrace{(3x^2-5x)}^{f(x)}}{h}$$

$$\frac{3(x^2+2xh+h^2)-5x-5h-3x^2+5x}{h}$$

$$\lim_{h\to 0} \frac{\cancel{3x^2}+6xh+3h^2-\cancel{5x}-5h-\cancel{3x^2}+\cancel{5x}}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(6x+3h-5)}{\cancel{h}} = 6x-5$$

$$\boxed{f'(x)=6x-5}$$

Find $f'(x)$.

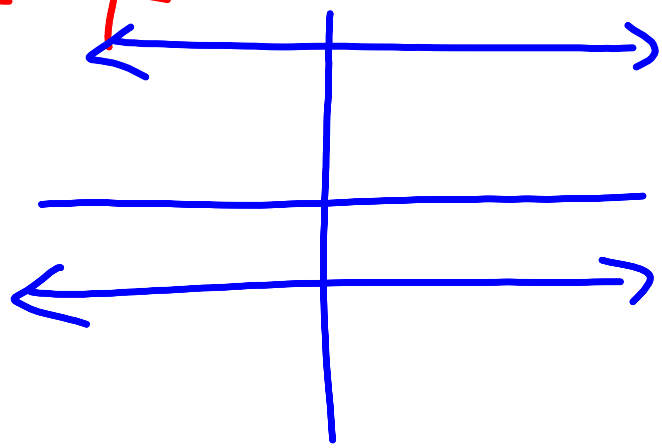
$$f(x)=2x^{3-1}+7x^{0}$$

Power Rule

$$f'(x)=6x^2+\underline{0}$$

$$f'(x)=6x^2$$

$y=7$



$y=-1$

Find $\frac{dy}{dx}$.

$$y = \sqrt{x+2}$$

$f(x) = 3x^2 - 5x$

$f'(x) = 6x - 5$

Find the ***EQUATION Of THE TANGENT LINE***

at $x = -1$.

1) Slope
2) point

$3(-1)^2 - 5(-1)$

$(-1, 8)$

$x_1 \quad y_1$

Slope $= m = f'(x) = f'(-1) = 6(-1) - 5 = -11$

$y - 8 = -11(x + (+1))$

$y - 8 = -11x - 11$

$+8 \qquad +8$

$y = -11x - 3$

$$y = 3/x$$

Find the equation for the tangent line at (1,3).

Graph 2.1 Limit Tangent Line.tii

Define Derivative & NDER.gsp