Example

$* \; 2x-3$

Find the equation of the line tangent to $f(x) = x^2 - 3x + 8$ at $x = 3$.

1. Slope $f'(3)=3$

2. point $(3, 8)$

$f'(x)=2x-3$

$f'(3)=2(3)-3=3$

$$\lim_{h\to 0} \frac{(x+h)^2-3(x+h)+8-(x^2-3x+8)}{h}$$

$$\frac{\cancel{x^2}+2xh+h^2-\cancel{3x}-3h+\cancel{8}-\cancel{x^2}+\cancel{3x}-\cancel{8}}{h}$$

$$\frac{2xh+h^2-3h}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(2x+h-3)}{\cancel{h}} = 2x+0-3$$

$$= 2x-3$$

$y-y_1=m(x-x_1)$ $(3, 8)$ $m=3$

$y-8=3(x-3)$

$y-8=3x-9$

$+8 \qquad +8$

$y=3x-1$ equation of tangent line

normal line = $\perp$ to tangent line

Find the equation of the ~~line tangent~~ normal line to $f(x) = x^2 - 3x + 8$ at $x = 3$.

1. slope $m = -1/3$
2. point $(3, 8)$

$f'(x) = 2x - 3$

$f'(3) = 3$ slope of tangent

$y - 8 = -\frac{1}{3}(x - 3)$

$y - \cancel{8} = -\frac{1}{3}x + 1$

$\quad + \cancel{8} \qquad\qquad + 8$

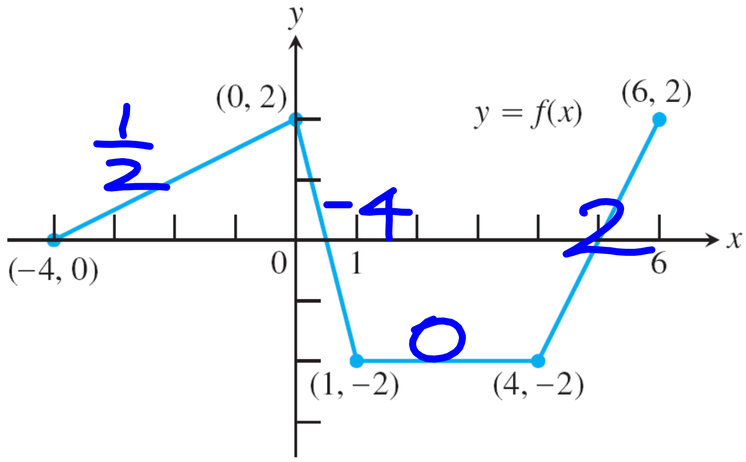
$y = -\frac{1}{3}x + 9$

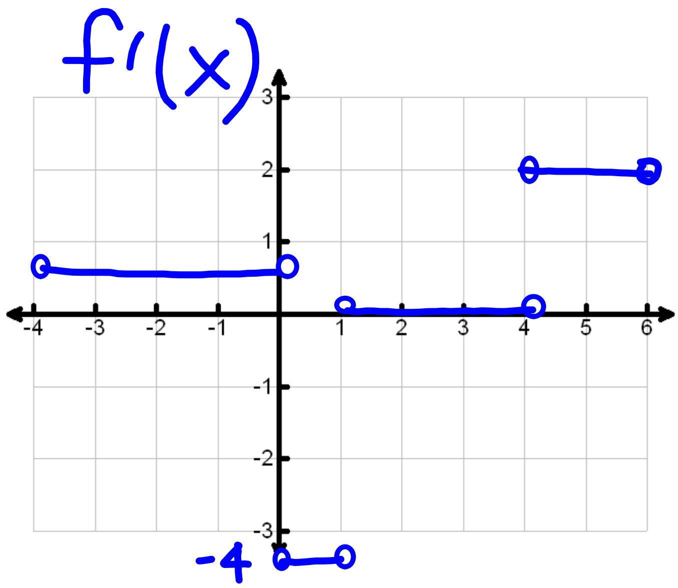
normal line

Example

Sketch a graph of $f'(x)$.

$f'(x) = \text{slope}$



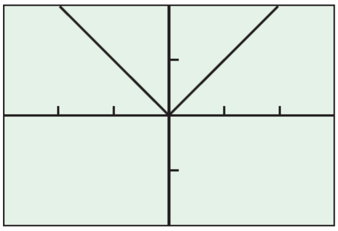


Graphs Differentiablity.tii

When is a function NOT differentiable?

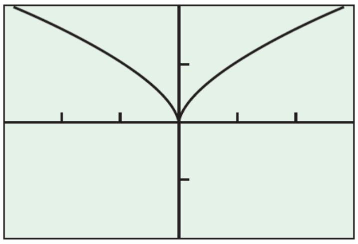
. . . when the limit of the difference quotient does not exist at a value $x = a$.

1. **Corner** (the one-sided ~~limits~~ slopes differ)



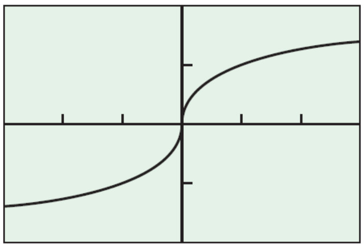
[–3, 3] by [–2, 2]

2. **Cusp** (the limit of the difference quotient approaches ∞ from one side and $-\infty$ from the other side)



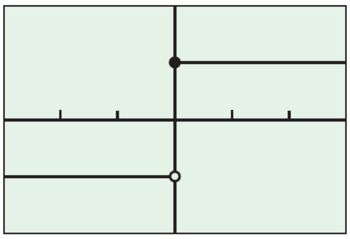
[–3, 3] by [–2, 2]

3. **Vertical Tangent** (the limit of the difference quotient approaches ∞ or $-\infty$ from both sides)



[–3, 3] by [–2, 2]

4. **Discontinuity** (one or both sides of the difference quotient will not exist)



[–3, 3] by [–2, 2]

* Look at a graph to determine differentiablity.

Example

Describe the x-values at which the function is
not
differentiable.

$$f(x) = |x^2 - 16|$$

$x = \pm 4$ corner

Example

Describe the x-values at which the function is differentiable.

not

$$f(x) = \frac{2x-4}{x+3}$$

$X = -3$ infinite discontinuity

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Solve for the derivative using the limit definition.

$$f(x) = x^2 + 5x + 6$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{(x+h)^2 + 5(x+h) + 6 - (x^2 + 5x + 6)}{h}$$

$$\frac{x^2 + 2xh + h^2 + 5x + 5h + 6 - x^2 - 5x - 6}{h}$$

$$\lim_{h \to 0} \frac{2xh + h^2 + 5h}{h} = \frac{\cancel{h}(2x + h + 5)}{\cancel{h}}$$

$$2x + 0 + 5 = 2x + 5$$

Attachments

Graphs Differentiablity.tii

Graph 2.1 Limit Tangent Line.tii