Conclusion about differentiablity:

Most functions in calculus are differentiable:

polynomial
rational
trigonometric
exponential
logarithmic
compositions of the above

If a function has a derivative at $x = a$, then it is continuous at $x = a$.

The converse IS NOT TRUE!

Warm-up: 1.28.20

1) Pick up handouts and glue pages into INB.

2) Get out HW and ck/calendar sheet.

3) Copy the following problem on pg. 29 in INB and complete. "**Write the equation of the tangent line to f(x)= $-2x^2$ at x = 2.**"

$y = -2x^2 \quad x = 2$

1. slope-derivative $f'(x) = -4x \quad -4(2)$

$m = -8$

2. point $(2, -8)$

$$\lim_{h \to 0} \frac{-2(x+h)^2 - (-2x^2)}{h}$$

$$\frac{-2(x^2+2xh+h^2) + 2x^2}{h}$$

$$\lim_{h \to 0} \frac{\cancel{-2x^2} - 4xh - 2h^2 + \cancel{2x^2}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(-4x - 2\cancel{h})}{\cancel{h}} = -4x$$

$m = -8 \quad pt\ (2, -8)$

$y - (-8) = -8(x-2)$

$y + 8 = -8x + 16$

$\quad -8 \qquad\qquad -8$

$y = -8x + 8$

$$f(x) = \frac{2}{x} = 2x^{-1}$$

$$f(x) = \sqrt[3]{x^2} = x^{2/3}$$

$$f(x) = x(\sqrt{x} + 3)$$

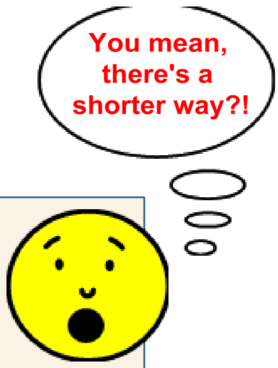
$$x^1(x^{\frac{1}{2}} + 3)$$

$$= x^{3/2} + 3x$$

$$4x^{3/2}$$

$$4\sqrt{x^3}$$

2.2 Basic Differentiation Rules



RULE 1 The Constant Rule $f(x)=5$

If f is the function with the constant value c, then

$y=5$ Hoy

$$\frac{df}{dx} = \frac{d}{dx}(c) = 0.$$

RULE 2 The Power Rule $f(x)=x^n$ $f'(x)=nx^{n-1}$

If n is a rational number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}.$$

RULE 3 The Constant Multiple Rule $f(x)=4x^3$

If u is a differentiable function of x and c is a constant, then

$4 \cdot f(x)=x^3$

$$\frac{d}{dx}(cu) = c\frac{du}{dx}.$$

$f'(x)=3 \cdot 4x^2$

$=12x^2$

RULE 4 **The Sum and Difference Rule**

If u and v are differentiable functions of x, then their sum and difference are differentiable at every point where u and v are differentiable. At such points,

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}.$$

$x^0 = 1$

$f(x) = 5x^{2-1} + x^{1-1} + 1$

$\boxed{f'(x) = 10x + 1}$

Example

$p' = 3t^2 + 12t - \frac{5}{3}$

Find $\frac{dp}{dt}$ if $p = t^3 + 6t^2 - \frac{5}{3}t' + 16.$

<u>Example</u> Find $f'(x)$.

$$f(x) = \frac{1}{x^2} = x^{-2-1}$$

$$f'(x) = -2x^{-3} = \frac{-2}{x^3}$$

Example Find $\frac{dy}{dx}$.

$f(x) = x^n$ $\quad f'(x) = nx^{n-1}$

$$y = \sqrt{x} - 4$$

$$y = x^{1/2} - 4 \qquad \tfrac{1}{2} - 1$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

Examples Find the derivative of each function.

1. $f(x) = 3\sqrt{x} + 8x = 3x^{\frac{1}{2}} + 8x$

$f'(x) = \frac{3}{2}x^{-1/2} + 8$

$\frac{x^2}{\cancel{x^3}} \quad x^2 \cdot x^{-3} = x^{-1}$ $\quad 3x^{\frac{1}{2}}$ $\quad 1-2 \quad \frac{1}{2}-2$

2. $f(x) = \frac{x + 3\sqrt{x}}{1x^2} = \frac{x + 3x^{\frac{1}{2}}}{x^2} = \frac{x^1}{x^2} + \frac{3x^{\frac{1}{2}}}{x^2}$

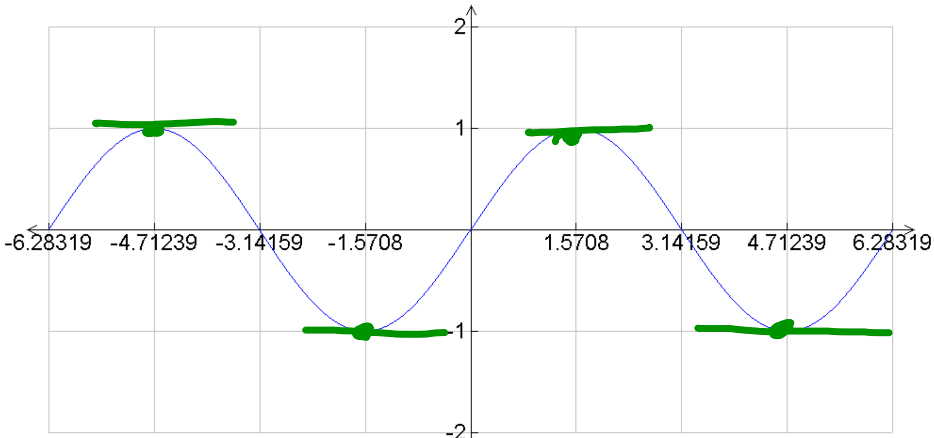
$= x^{-1} + 3x^{-3/2}$ $\quad -\frac{2}{2}$ $\quad \boxed{f'(x) = -x^{-2} - \frac{9}{2}x^{\frac{-5}{2}}}$

3. $f(x) = \frac{5}{x^{2/3}} = 5x^{-2/3}$ $\quad -\frac{3}{3}$ $\quad f'(x) = -\frac{10}{3}x^{-\frac{5}{3}}$

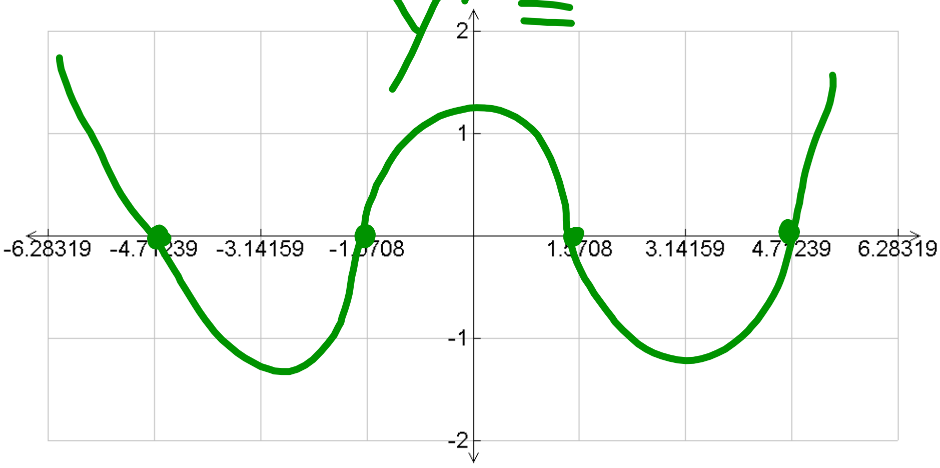
$-\frac{2}{3} \cdot \frac{5}{1}$

What is the derivative of sin*x*?

$y = \sin x$



$y' =$



The Derivatives of Sine and Cosine

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

Memorize These!

More Trig Derivatives

$$\frac{d}{dx}\tan x = \sec^2 x, \qquad \frac{d}{dx}\sec x = \sec x \tan x$$
$$\frac{d}{dx}\cot x = -\csc^2 x, \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

<u>Example</u> Find the derivative.

$$f(x) = \frac{\pi}{2}\sin x - \cos x \qquad f'(x) = \frac{\pi}{2}\cos x - (-\sin x)$$

$$\boxed{\pi/2 \cos x + \sin x}$$

$\frac{\pi}{2} f(x) = \sin x$

$f'(x) = \frac{\pi}{2}\cos x$

$f(x) = -\cos x$

$-f(x) = \cos x$

$-f'(x) = -\sin x$

$f'(x) = \sin x$

Horizontal Tangents

Example Determine the point(s) at which the function has a horizontal tangent.

$$\boxed{y = 3x^4 - 11x^2 - 4}$$

$y' = 12x^3 - 22x$

$0 = 12x^3 - 22x$

$0 = 2x(6x^2 - 11)$

$2x = 0$

$x = 0$

$6x^2 - 11 = 0$

$+11 \quad +11$

$\frac{\cancel{6}x^2}{\cancel{6}} = \frac{11}{6}$

$\sqrt{x^2} = \sqrt{\frac{11}{6}}$

$x = \pm\sqrt{\frac{11}{6}}$

$$y = \frac{5}{3x^3} = \frac{5}{3}x^{-3}$$