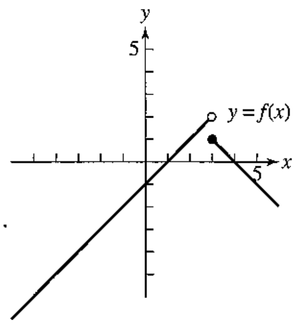
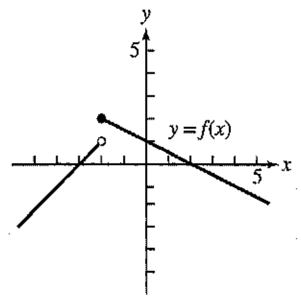
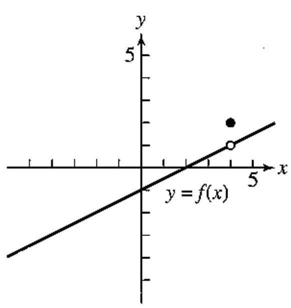
41. One possible answer:



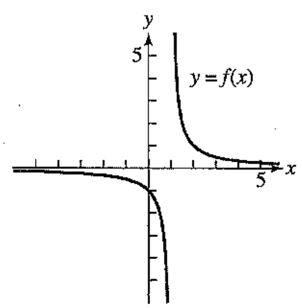
42. One possible answer:



43. One possible answer:

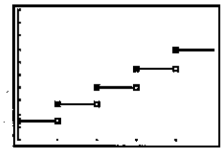


44. One possible answer:



51. Consider $f(x) = x - e^{-x}$, f is continuous, $f(0) = -1$, and $f(1) = 1 - \frac{1}{e} > 0.5$. By the Intermediate Value Theorem, for some c in $(0, 1)$, $f(c) = 0$ and $e^{-c} = c$.

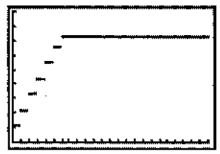
52. (b)



[0, 4.8] by [35000, 45000]

Continuous at all points in the domain [0, 5) except at $t = 1, 2, 3, 4$.

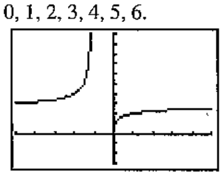
53. (b)



[0, 24] by [0, 9]

This is continuous at all values of x in the domain [0, 24] except for $x = 0, 1, 2, 3, 4, 5, 6$.

60. (b)



[–5, 5] by [–3, 10]

(c) Because f is undefined there due to division by 0.

(d) $x = 0$: removable, right-hand limit is 1
$x = -1$; not removable, infinite discontinuity

(e) 2.718 or e

61. This is because $\lim_{h \to 0} f(a + h) = \lim_{x \to a} f(x)$.

62. Suppose not. Then f would be negative somewhere in the interval and positive somewhere else in the interval. So, by the Intermediate Value Theorem, it would have to be zero somewhere in the interval, which contradicts the hypothesis.

63. Since the absolute value function is continuous, this follows from the theorem about continuity of composite functions.

64. For any real number a, the limit of this function as x approaches a cannot exist. This is because as x approaches a, the values of the function will continually oscillate between 0 and 1.

Section 2.4

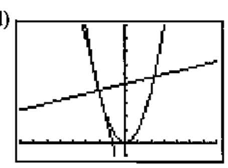
Exercises 2.4

9. (a) -4

(b) $y = -4x - 4$

(c) $y = \frac{1}{4}x + \frac{9}{2}$

(d)



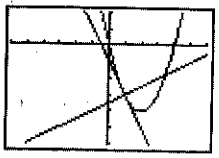
[–8, 7] by [–1, 9]

10. (a) -2

(b) $y = -2x - 1$

(c) $y = \frac{1}{2}x - \frac{7}{2}$

(d)

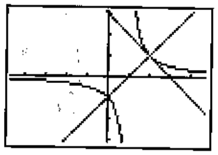


[−6, 6] by [−6, 2]

11. **(a)** −1 **(b)** $y = -x + 3$

(c) $y = x - 1$

(d)

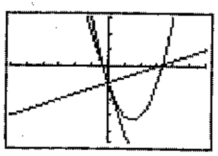


[−4.7, 4.7] by [−3.1, 3.1]

12. **(a)** −3 **(b)** $y = -3x - 1$

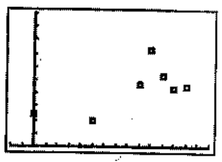
(c) $y = \frac{1}{3}x - 1$

(d)



[−6, 6] by [−5, 3]

34. **(a)**



[−2, 15] by [0, 50]

47. This function has a tangent with slope zero at the origin. It is sandwiched between two functions, $y = x^2$ and $y = -x^2$, both of which have slope zero at the origin.

Looking at the difference quotient, $-h \le \frac{f(0+h) - f(0)}{h} \le h$, so the Sandwich Theorem tells us that the limit is 0.

48. This function does not have a tangent line at the origin. As the function oscillates between $y = x$ and $y = -x$ infinitely often near the origin, there are an infinite number of difference quotients (secant line slopes) with a value of 1 and with a value of −1. Thus the limit of the difference quotient doesn't exist.

The difference quotient is $\frac{f(0+h) - f(0)}{h} = \sin\left(\frac{1}{h}\right)$ which oscillates between 1 and −1 infinitely often near zero.

Review Exercises

27. **(a)** Vertical Asymp.: $x = -2$

(b) Left-hand limit $= -\infty$

Right-hand limit $= \infty$

28. **(a)** Vertical Asymp.: $x = 0$ and $x = -2$

(b) At $x = 0$:

Left-hand limit $= -\infty$

Right-hand limit $= -\infty$

At $x = -2$:

Left-hand limit $= -\infty$

Right-hand limit $= -\infty$

29. **(a)** At $x = -1$:

Left-hand limit = 1

Right-hand limit = 1

At $x = 0$:

Left-hand limit = 0

Right-hand limit = 0

At $x = 1$:

Left-hand limit = −1

Right-hand limit = 1

(b) At $x = -1$:

Yes, the limit is 1.

At $x = 0$:

Yes, the limit is 0.

At $x = 1$:

No, the limit doesn't exist because the two one-sided lim different.

(c) At $x = -1$:

Continuous because $f(-1)$ = the limit.

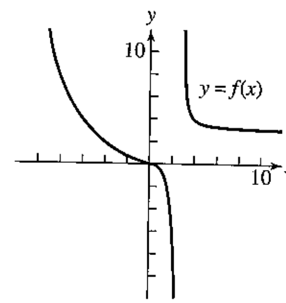
At $x = 0$:

Discontinuous because $f(0) \ne$ the limit.

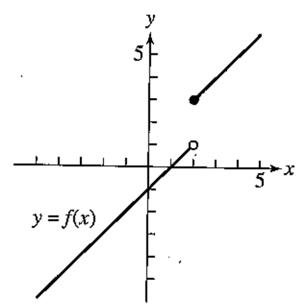
At $x = 1$:

Discontinuous because limit doesn't exist.

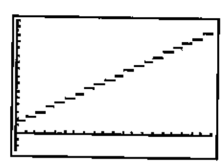
41. One possible answer:



42. One possible answer:

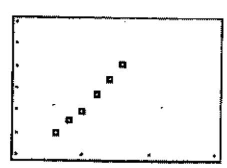


50. **(a)** $\begin{cases} f(x)3.20 - 1.35 \cdot \text{int}(-x+1), & 0 < x \le 20 \\ 0, & x = 0 \end{cases}$



[0, 20] by [−5, 32]

51. **(a)**



[5, 20] by [15000, 18000]

ck Review 3.1 *(For help, go to Sections 2.1 and 2.4.)*

cises 1–4, evaluate the indicated limit algebraically.

$\frac{(2 + h)^2 - 4}{h}$ 4 **2.** $\lim_{x \to 2^+} \frac{x + 3}{2}$ 5/2

$\frac{|y|}{y}$ −1 **4.** $\lim_{x \to 4} \frac{2x - 8}{\sqrt{x} - 2}$ 8

d the slope of the line tangent to the parabola $y = x^2 + 1$ s vertex. 0.

considering the graph of $f(x) = x^3 - 3x^2 + 2$, find the rvals on which f is increasing. $(-\infty, 0]$ and $[2, \infty)$

In Exercises 7–10, let

$$f(x) = \begin{cases} x + 2, & x \le 1 \\ (x - 1)^2, & x > 1. \end{cases}$$

7. Find $\lim_{x \to 1^+} f(x)$ and $\lim_{x \to 1^-} f(x)$. $\lim_{x \to 1^+} f(x) = 0$; $\lim_{x \to 1^-} f(x) = 3$

8. Find $\lim_{h \to 0^+} f(1 + h)$. 0

9. Does $\lim_{x \to 1} f(x)$ exist? Explain. No, the two one-sided limits are different.

10. Is f continuous? Explain. No, f is discontinuous at $x = 1$ because the limit doesn't exist there.

ion 3.1 Exercises

ises 1–4, use the definition

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

ie derivative of the given function at the indicated point.

$= 1/x, a = 2$ −1/4 **2.** $f(x) = x^2 + 4, a = 1$ 2

$= 3 - x^2, a = -1$ 2 **4.** $f(x) = x^3 + x, a = 0$ 1

ses 5–8, use the definition

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

e derivative of the given function at the indicated point.

$= 1/x, a = 2$ −1/4 **6.** $f(x) = x^2 + 4, a = 1$ 2

$= \sqrt{x + 1}, a = 3$ 1/4 **8.** $f(x) = 2x + 3, a = -1$ 2

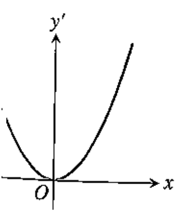
$f'(x)$ if $f(x) = 3x - 12$. $f'(x) = 3$

dy/dx if $y = 7x$. $dy/dx = 7$

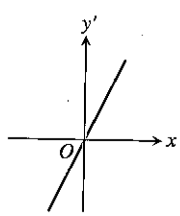
$\frac{d}{dx}(x^2)$. $2x$

$\frac{d}{dx} f(x)$ if $f(x) = 3x^2$. $6x$

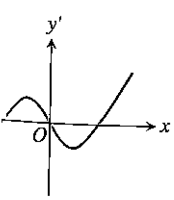
ses 13–16, match the graph of the function with the graph of tive shown here:



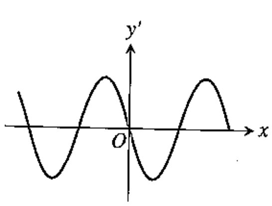
(a)



(b)

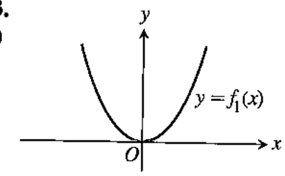


(c)

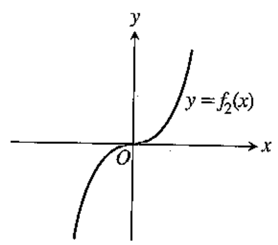


(d)

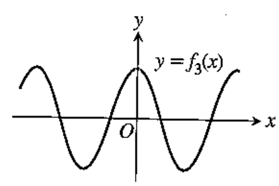
13. (b)



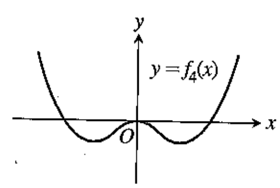
14. (a)



15. (d)



16. (c)



17. If $f(2) = 3$ and $f'(2) = 5$, find an equation of **(a)** the *tangent* line, and **(b)** the *normal* line to the graph of $y = f(x)$ at the point where $x = 2$.

(a) $y = 5x - 7$

(b) $y = -\frac{1}{5}x + \frac{17}{5}$