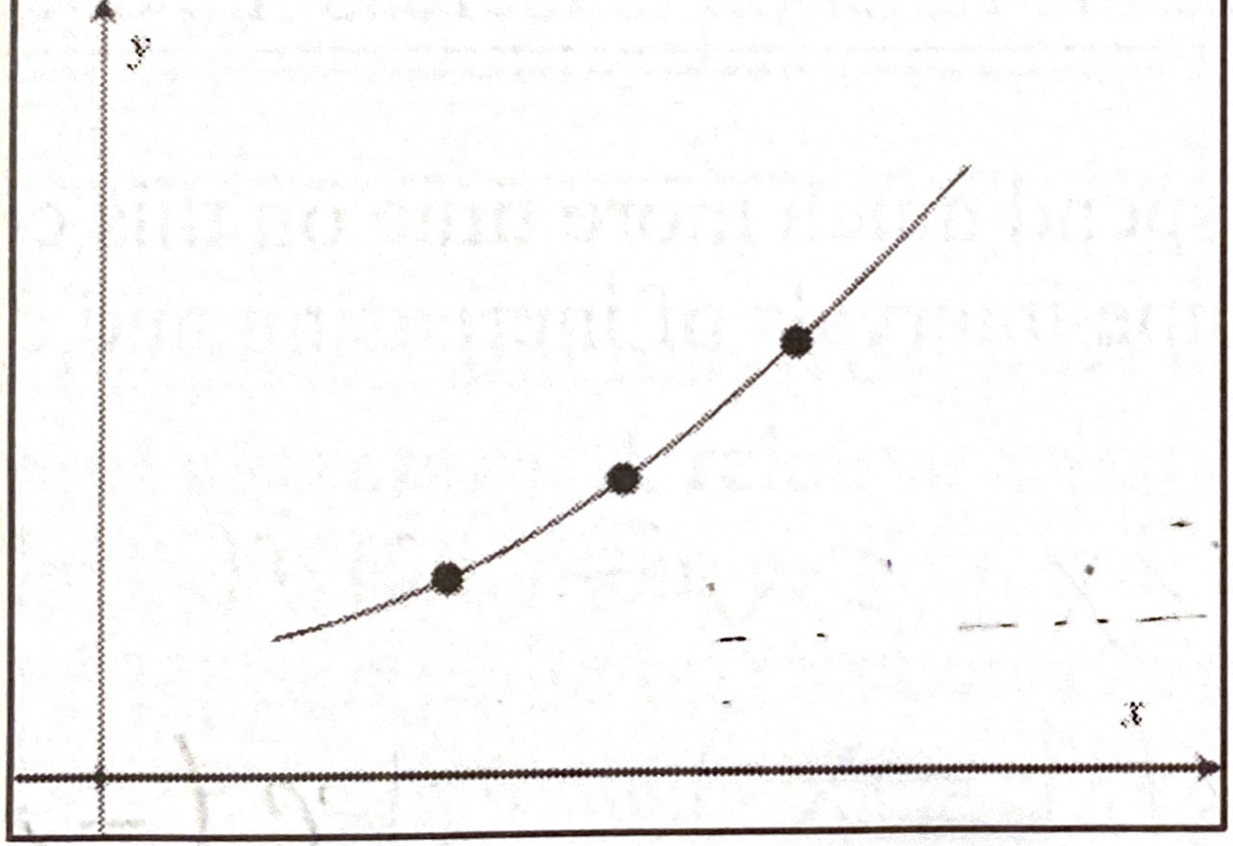
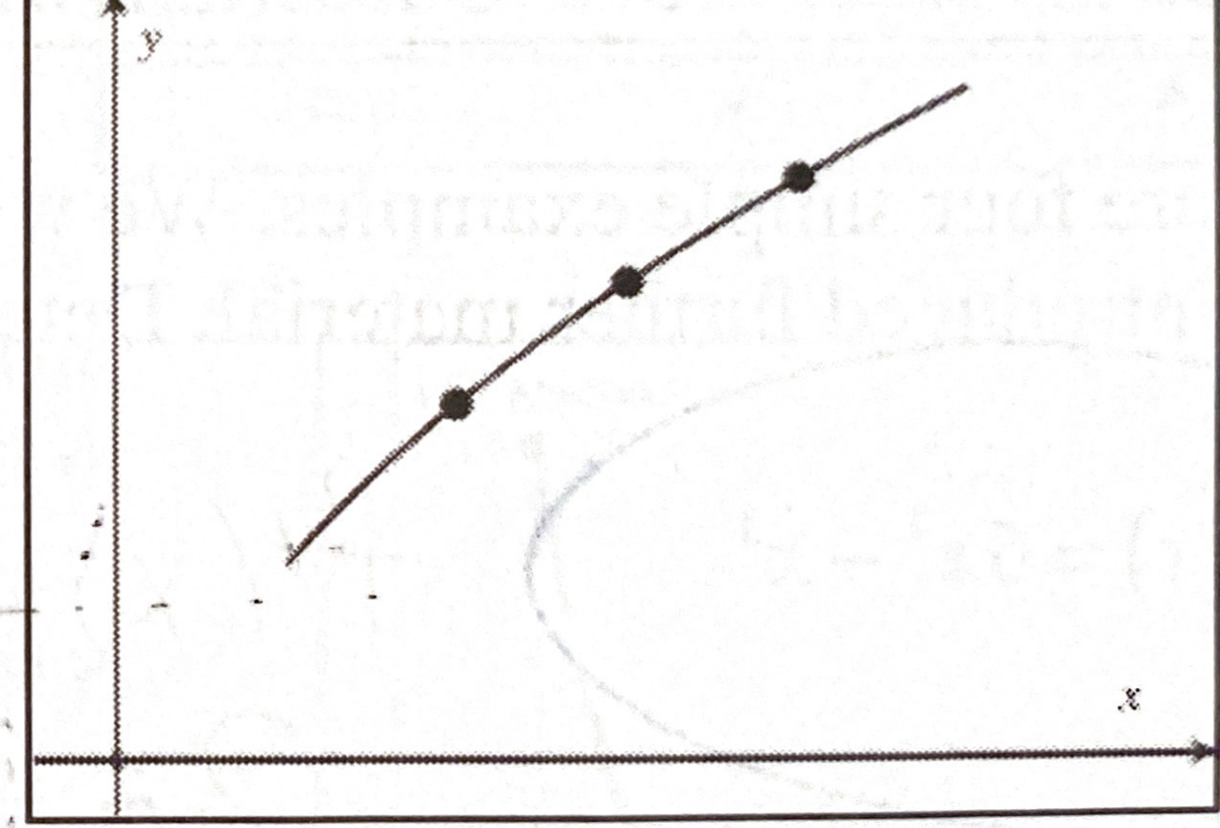
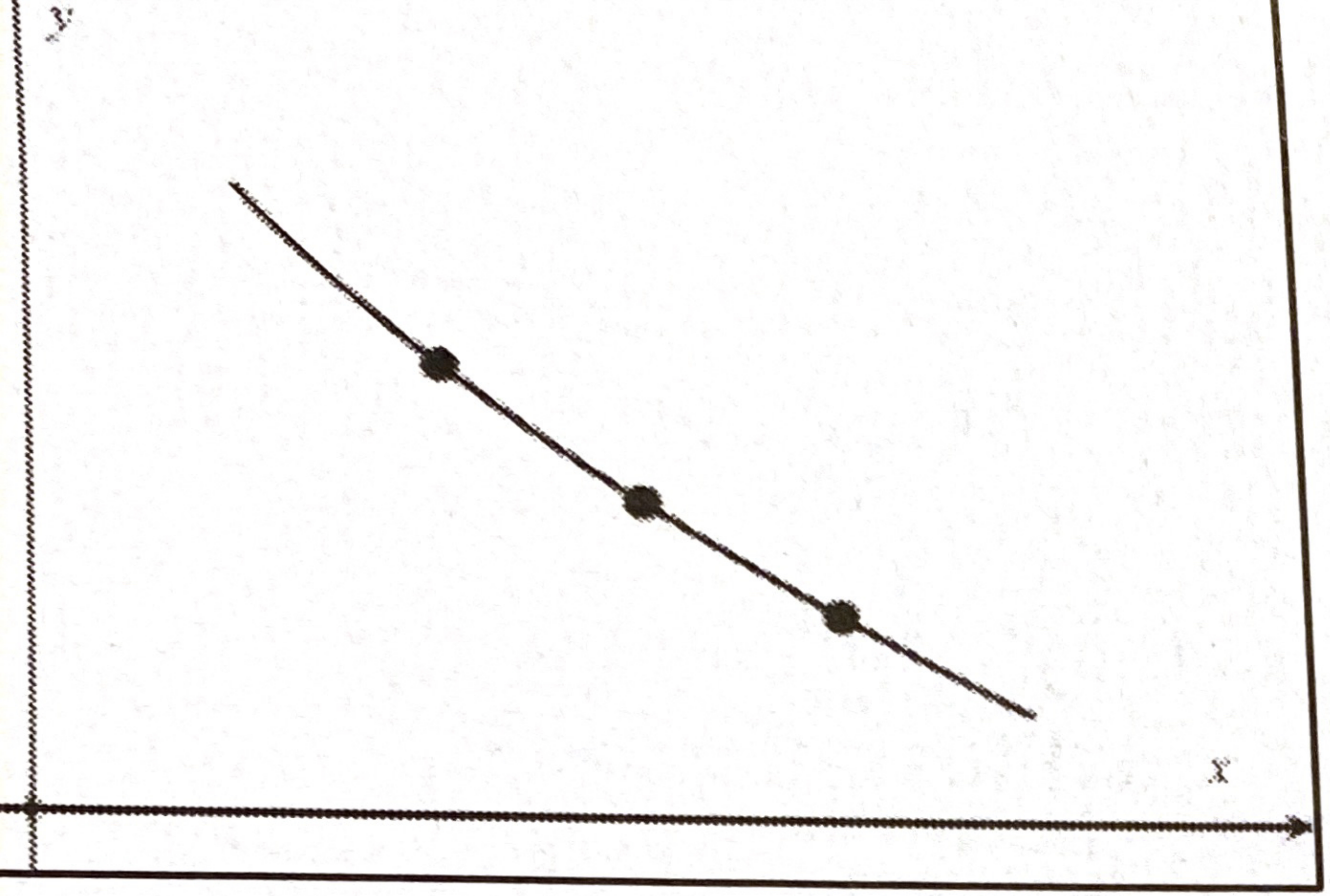
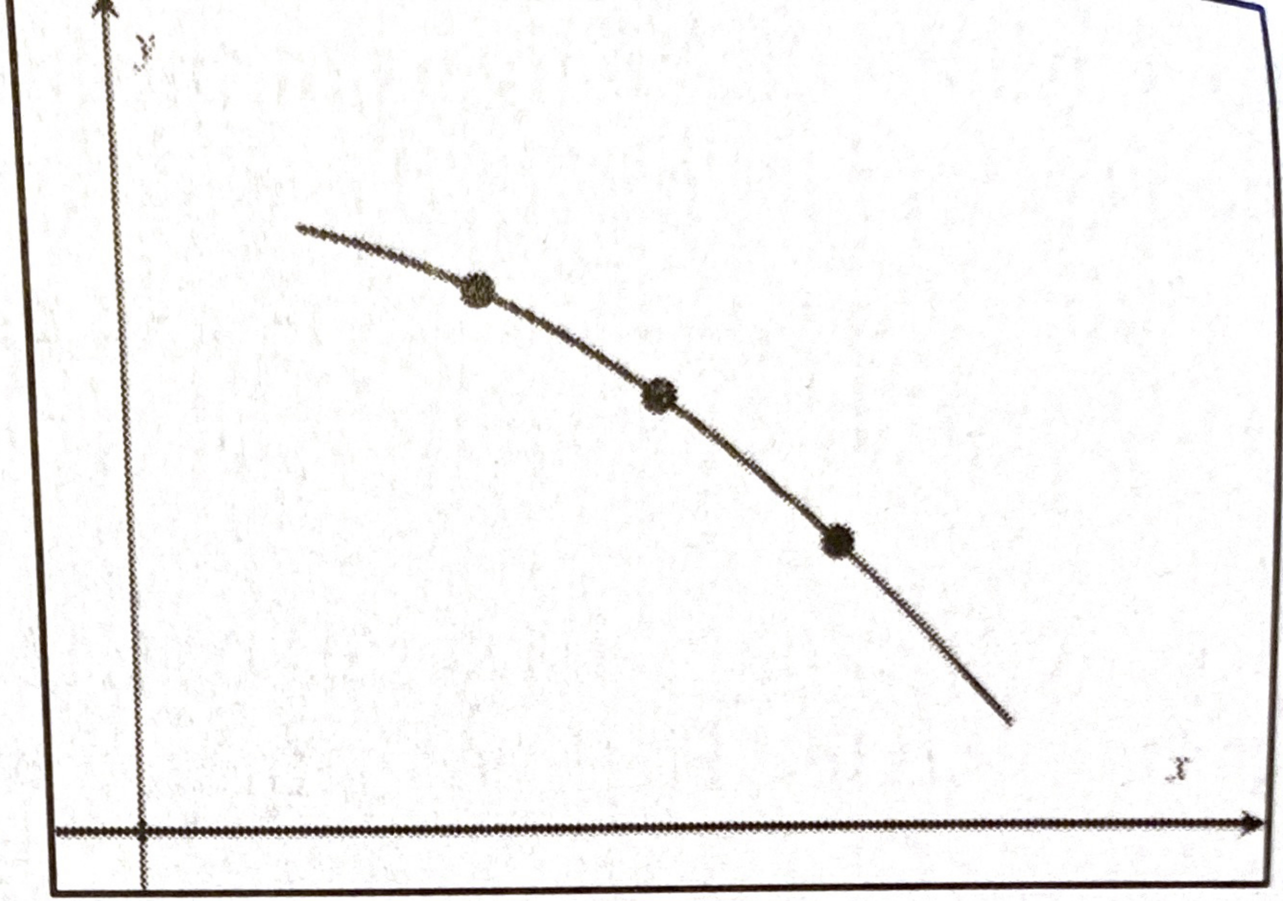




ines (the derivative) in increasing curves? positive





es (the derivative) in decreasing curves? negative

about increasing and decreasing functions. Let f be a function that
nd differentiable on the open interval (a,b).

1. Locate the critical values of f on its domain. Do this by finding $f'(x)$ and determining values of x where $f'(x)=0$ (also called a **stationary point**) or $f'(x)$ does not exist. Usually you will have to have $f(x)$ in fractional form with both numerator and denominator factored, if possible.
2. Make a sign chart placing all critical values on the x-axis, creating intervals.
3. Use a test number in the interval to determine the sign of $f'(x)$ in that interval.
4. Use the chart above to ascertain where the function is increasing, decreasing or constant.

Here are four simple examples. We will spend much more time on this concept later in this section when we have introduced further material. Determine intervals of increasing and decreasing for:

1) $f(x)=6x^2-x^3$

$f'(x)=12x-3x^2$

$0=3x(4-x)$

$3x=0$ $\quad$ $4-x=0$

$x=0$ $\quad$ $x=4$

dec $(-\infty,0)\cup(4,\infty)$

inc $(0,4)$

$-$ $\quad$ $+$ $\quad$ $-$

0 $\quad$ 4

$f'(-1)$ $\quad$ $f'(1)$ $\quad$ $f'(5)$

2) $f(x)=(x-2)^2(x+1)$

5) Find relative extrema of $f(x) = 5x^4 - 20x^2$.

$f'(x) = 20x^3 - 40x$

$20x(x^2 - 2) = 0$

$20x = 0$

$x = 0$

$x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

$-$ $+$ $-$ $+$

$-\sqrt{2}$ 0 $\sqrt{2}$ $f'(x)$

6) Find relative extrema of $f(x) = 5x^4 - 20x^3$.

at $x = -\sqrt{2}$ it changes from dec to inc so it is a rel/abs min

at $x = 0$ it changes from inc to dec so it is a rel/abs max

at $x = \sqrt{2}$ it changes from dec to inc so it is a rel/abs min

7) Find relative extrema of the function

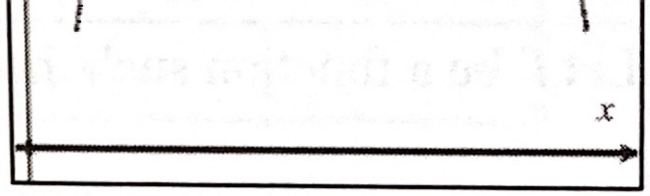
$f(x) = \cos x + \frac{1}{2}x$ on $[0, 2\pi]$.

8) Find relative extrema of $f(x) = -(36 - x^2)^{2/3}$.

* rel/abs min @ $(-\sqrt{2}, -20)$ and $(\sqrt{2}, -20)$
* rel max @ $(0, 0)$

So we defin

$f''(x)$ changes from positive to negative or negative to positive. In the figure to the right, $f'(x)=0$ at these inflection points. When that occurs, we called them **stationary inflection points.**



Determine intervals where $f(x)$ is concave up, concave down, and any inflection points. Confirm graphically.

9) $f(x)=x^3-3x^2+3$ $\quad f'(x)=3x^2-6x \quad f''(x)=6x-6$

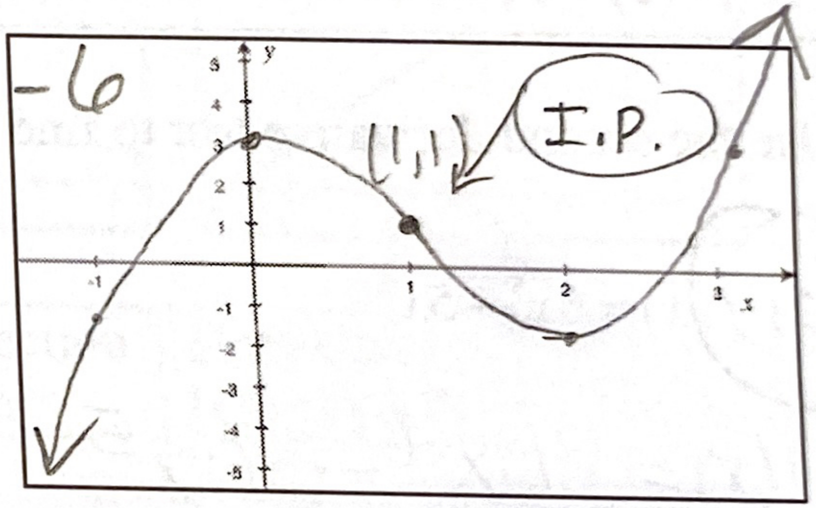
$6x-6=0$

$x=1$

$f''(x)$: $-$ on $(-\infty, 1)$, $+$ on $(1, \infty)$

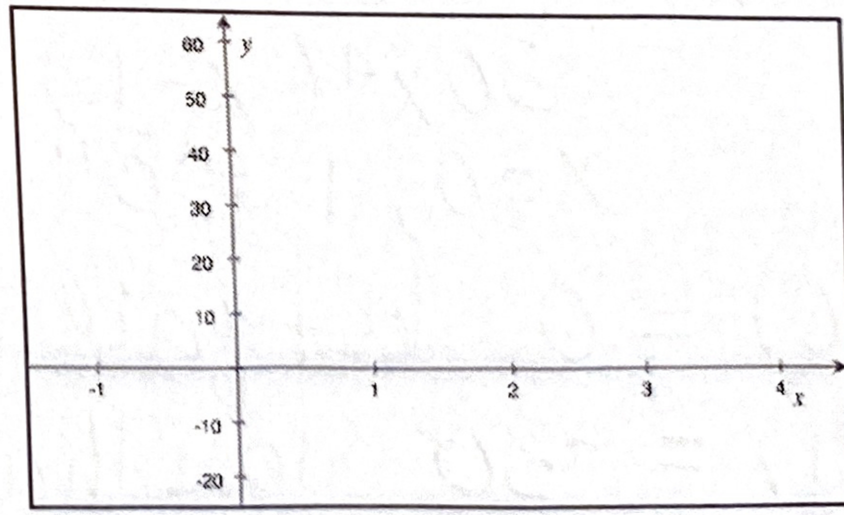
CD $(-\infty, 1)$ $\quad$ CU $(1, \infty)$

I.P. @ $(1, 1)$



Note: In this case, the inflection point occurs at the point between the relative maximum and relative minimum where the curve is the steepest. If you were running up this hill, it would be the most difficult location to run. This inflection point is not a stationary inflection point as $f'(1)\neq 0$.

10) $f(x)=8x^3-2x^4+3$



- If $f''(c)>0$, then f has a relative minimum at $(c, f(c))$.
- If $f''(c)<0$, then f has a relative maximum at $(c, f(c))$.
- If $f''(c)=0$, then the test fails and you must use the first derivative test to ascertain relative extrema.

Use the second derivative test to find relative extrema for:

12) $f(x)=6x^5-5x^6$

13) $f(x)=\sqrt{x^2+4}$

14) $f(x)=\sin x\cos x \quad \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$f'(x)=30x^4-30x^5$

$f''(x)=120x^3-150x^4$

rel max @ (1, 1)

$f'(x)=0=30x^4-30x^5$

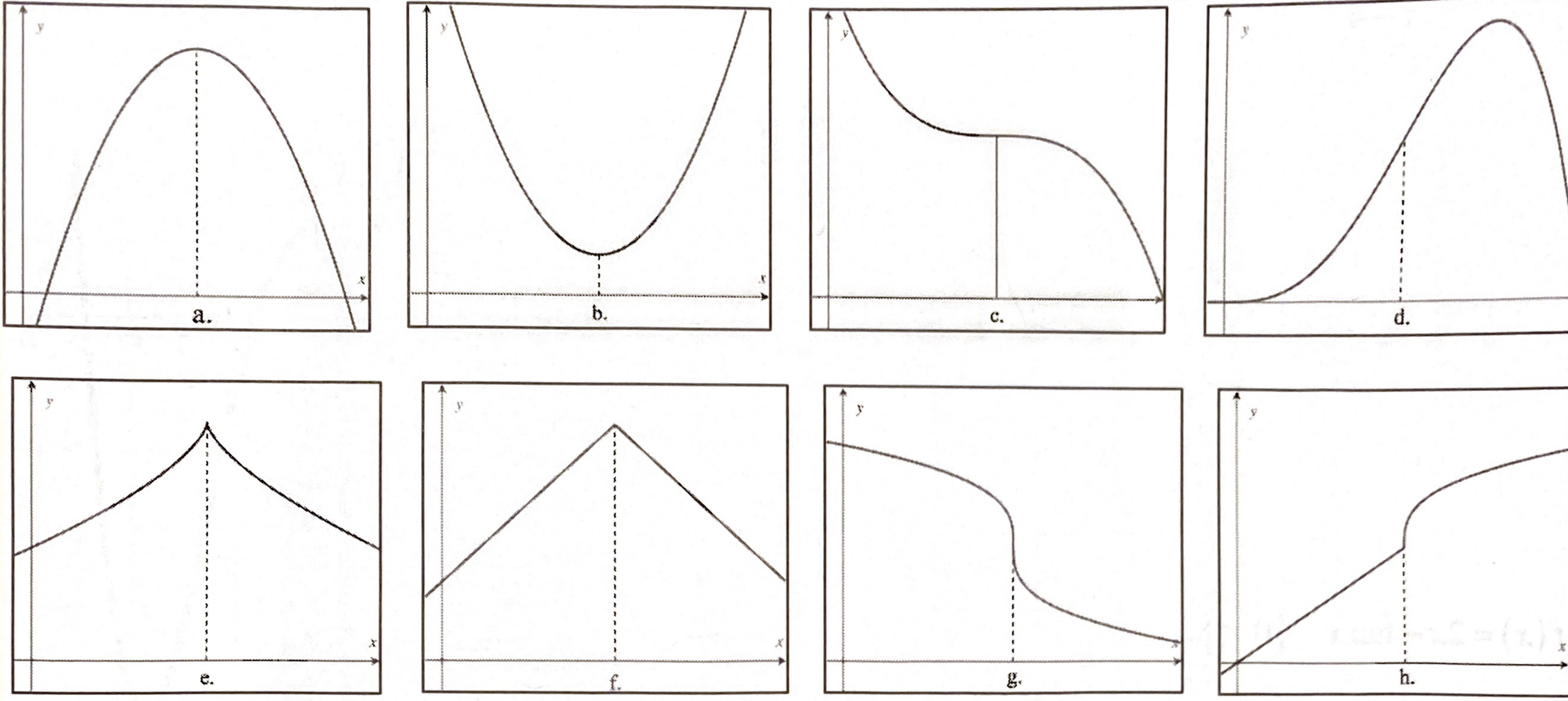
$30x^4(x-1)$

$x=0, 1$ = "c"

ck graph to confirm

$f''(0)=0$ test fails

$f''(1)=-30$ less than zero so it is a rel max



	Critical Value	Stationary Point $f'(x)=0$	Relative Minimum	Relative Maximum	Inflection Point	Stationary Inflection Pt.
a.	✓	✓		✓		
b.	✓	✓	✓			
c.	✓	✓			✓	✓
d.					✓	
e.	✓	✓		✓		
f.	✓	✓		✓		
g.	✓	✓			✓	✓
h.	✓	✓			✓	✓

ditional calculus courses require students to graph functions, given its equation. With the advent of graph ulators and technology, there is now less emphasis on actually graphing functions and more emphasis on cribing graphs in terms of increasing, decreasing, relative extrema, concavity and inflection points. This ual will use that approach and not actually expend energy on graphing which is extremely time-consumi