Algebra I

Unit 1 Notes

Relationships between Quantities and Expressions

Name ___________________________

Standard:	Learning Objective:	What am I learning?	Mastery?
MGSE9–12.N.Q.1 Use units of measure (linear, area, capacity, rates, and time) as a way to understand problems: 1. Identify, use, and record appropriate units of measure within context, within data displays, and on graphs; 2. Convert units and rates using dimensional analysis (English–to–English and Metric– to–Metric without conversion factor provided and between English and Metric with conversion factor); 3. Use units within multi–step problems and formulas; interpret units of input and resulting units of output.	1.1	How to convert units between English to English	
	1.2	How to convert units between Metric to Metric	
	1.3	How to convert units between English to Metric	
	1.4	How to convert and use rates	
MGSE9–12.N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. Given a situation, context, or problem, students will determine, identify, and use appropriate quantities for representing the situation.	1.5	How to use appropriate units for measure (Example: using yards to measure a football field versus inches)	
MGSE9–12.N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	1.6	How to estimate appropriately for scenarios (Example: money should be estimated to the nearest hundredth or cent value; round to the whole number for objects)	
MGSE9–12.A.SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients, in context.	1.7	How to interpret parts of an expression such as terms, like terms, factors, coefficients, constants, and variables	
MGSE9–12.A.SSE.1b Given situations, which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors.	1.8	How to interpret parts of an expression in the context of a word problem	
MGSE9–12.N.RN.2 Rewrite expressions involving radicals (i.e., simplify and/or use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots).	1.9	How to simplify radicals using prime factors	
	1.10	How to multiply radicals	
	1.11	How to add and subtract radicals using like radicands	
MGSE9–12.A.APR.1 Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations. *(For the purpose of this course, operations with polynomials will be limited to the second degree.)*	1.12	How to add polynomials using like terms	
	1.13	How to subtract polynomials using the distributive property and like terms	
	1.14	How to multiply polynomials using exponent properties and like terms	
MGSE9–12.N.RN.3 Explain why the sum or product of rational numbers is rational; why the sum of a rational number and an irrational number is irrational; and why the product of a nonzero rational number and an irrational number is irrational.	1.15	How to describe the characteristics and differences of rational and irrational numbers	
	1.16	How to describe different sums using combinations of rational and irrational numbers	
	1.17	How to describe different products using combinations of rational and irrational numbers	

Algebraic Expressions Vocabulary

ALGEBRA TERM AND DEFINITION	EXAMPLES
______________________________ A mathematical statement with variables, numbers, addition, subtraction, multiplication, division, parenthesis, square roots, exponents...	
______________________________ Symbols or letters used to represent an unknown	
______________________________ Items that are being added, subtracted, or divided	
______________________________ A term with the same variable raised to the same power	
______________________________ The number in front of a variable. It can be ______________ or _______________	
______________________________ The number up in the air next to a base. It tells you the number of times you multiply something by itself.	
______________________________ What the exponent sits on. You cannot have a base without an exponent. It is the part of the expression that has been raised to a power.	
______________________________ A number that has no variable. It can be ______________ or _______________	
______________________________ Items that are being multiplied together. These can be numbers, variables, expressions in parentheses.	

Operational Words

Addition (+)	Multiplication (▪)	Exponents (x^n)
Subtraction (-)	Division (÷)	Square-Root (√)

Examples

A. The sum of a number and 10

____

B. The product of 9 and x squared

____

C. 9 less than g to the fourth power

____

D. 8 + 3x

____

Key Concepts

- Expressions are made up of **terms**. A term is a number, a variable, or the product of a number and variable(s). An addition or subtraction sign separates each term of an expression.
- In the expression $4x^2 + 3x + 7$, there are 3 terms: $4x^2$, $3x$, and 7.
- The **factors** of each term are the numbers or expressions that when multiplied produce a given product. In the example above, the factors of $4x^2$ are 4 and x^2. The factors of $3x$ are 3 and x.
- 4 is also known as the **coefficient** of the term $4x^2$. A coefficient is the number multiplied by a variable in an algebraic expression. The coefficient of $3x$ is 3.
- The term $4x^2$ also has an **exponent**. Exponents indicate the number of times a factor is being multiplied by itself. In this term, 2 is the exponent and indicates that x is multiplied by itself 2 times.
- Terms that do not contain a variable are called **constants** because the quantity does not change. In this example, 7 is a constant.

Expression	$4x^2 + 3x + 7$		
Terms	$4x^2$	$3x$	7
Factors	4 and x^2	3 and x	
Coefficients	4	3	
Constants			7

Examples	**$6x^3 — 4xy + 7x^2 — 12$**	**$3a^2b — 16abc + 8.5$**
Put the expression in descending order.		**ALREADY IN DESCENDING ORDER**
How many terms are there?		
Name the terms:		
Name the factors:		
Name the coefficient(s):		
Name the constant(s):		

You are buying 4 cokes a "d" dollars each. Tax is an additional $.58.

Write an expression for this situation.

How many terms are there?

Name the terms.

Name the factors.

Name the coefficients.

Name the constant.

Unit Conversions

VOCABULARY

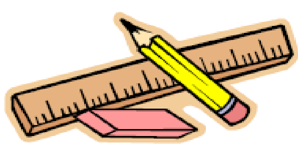
Unit Conversion - the act of changing the unit of measure, for instance changing 24 inches to 2 feet.

Dimensional Analysis - a process of converting units by using the fact any number or expression can be multiplied by 1 without changing its value.

Find the value of the following expressions.

1. $\frac{8}{8}$ 2. $\frac{x}{x}$ 3. $\frac{3y}{3y}$ 4. $\frac{feet}{feet}$ all equal… ______!

What happens when we divide something by itself?



Length

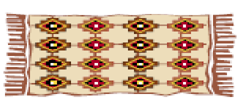
1 foot (ft) = 12 inches (in)

1 yard (yd) = 36 inches

1 yard = 3 feet

1 mile (mi) = 5,280 feet

1 mile = 1,760 yards



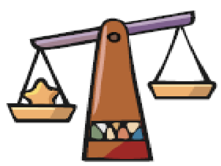
Area

1 square foot (ft^2) = 144 square inches (in^2)

1 square yard (yd^2) = 9 square feet

1 acre = 43,560 square feet

1 square mile (mi^2) = 640 acres



Mass/Weight

1 pound (lb) = 16 ounces (oz)

1 ton (T) = 2,000 pounds

Time

1 minute (min) = 60 seconds (s)

1 hour (h) = 60 minutes

1 day (d) = 24 hours

1 week (wk) = 7 days

1 year (y) = 12 months (mo)

~~1 year = 52 weeks~~

1 year = 365 days

1 decade = 10 years

1 century (c) = 100 years

1 millennium = 1,000 years

Capacity

1 cup (c) = 8 fluid ounces (fl oz)

1 pint (pt) = 2 cups

1 quart (qt) = 2 pints

1 quart (qt) = 4 cups

1 gallon (gal) = 4 quarts

Standard Units

Sometimes, the information that we are given in a problem is in the wrong *format/unit*.

For instance, we may be given a measurement in feet but be asked to solve a problem about miles. In this case, we need to ***convert*** the feet to miles before we can solve the problem.

Step 1: Write your path

Step 2: Write the proportion(s)/conversions that make your path *(Use as many unit conversions as it takes to get from one unit to the final. Sometimes it will be one; often it will be more.)*

Step 3: Write your units first (***no numbers***)

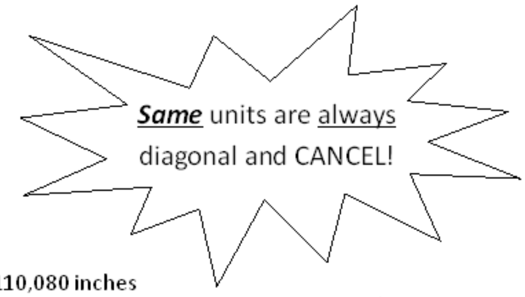
Step 4: Match the numbers with the units

Step 5: Multiply the numbers on top and bottom, then SIMPLIFY

***Example*:**

Convert 128 miles into inches.

Start: miles **End**: inches **Path**: miles —> *feet* —> inches



128 ~~miles~~	5280 ~~Feet~~	12 Inches	8,110,080 inches
1	1 ~~Mile~~	1 ~~Feet~~	1

= 8,110,080 inches

Ex 2) Convert 512 seconds to minutes.

____ =

Ex 3) Convert 4.8 pounds to ounces.

____ =

Ex 4) Convert 15 cups to quarts.

____ =

Ex 5) Convert 10.2 cups to quarts.

____ =

Ex 6) Convert 6.3 yards to inches.

____ =

Metric Unit Conversions

Conversions on the metric chart are all powers of _____.

<u>1</u> LARGE unit = _______ smaller units

"n" is the number of prefixes to get from the smaller to the bigger unit.

Example: 1 kg = _____ cg = _______________ cg

King	**H**enry	**D**ied	**U**nexpectedly	**D**rinking	**C**hocolate	**M**ilk
Kilo 10 x 10 x 10 x **<u>LARGER</u>** than the unit 1 kilo = 1,000 units	*Hecto* 10 x 10 x **<u>LARGER</u>** than the unit 1 hecto = 100 units	*Deca* 10 x **<u>LARGER</u>** than the unit 1 deca = 10 units	**Unit** ***Meter*** *(length)* ***Liter*** *(liquid volume)* ***Gram*** *(mass/weight)* **1 unit**	*Deci* 10 x **<u>SMALLER</u>** than the unit 10 deci = 1 unit	*Centi* 10 x 10 x **<u>SMALLER</u>** than the unit 100 centi = 1 unit	*Milli* 10 x 10 x 10 x **<u>SMALLER</u>** than the unit 1,000 milli = 1 unit
5 kilo	50 hecto	500 deca	5,000 units	50,000 deci	500,000 centi	5,000,000 milli

Example 1: Convert from 122 cL to kL

khdBdcm _____ *kL =* __________ *cL*

Example 2: Convert from 45 g to mg

khdBdcm _____ *g =* ___________ *mg*

Example 3: Convert from 4200 dm to hm

khdBdcm _____ *hm =* __________ *dm*

Example 4: Convert from 4.32 dag to mg

khdBdcm _____ *dag =* _________ *mg*

VOCABULARY

Rate - a unit of measure that includes both an amount and a time frame. For instance, miles per hour OR words per minute.

Advanced Unit Conversions

Sometimes it is necessary to go between different types of measurement (English to metric). In these problems the unit conversion will be given to you.

Ex. 1 Convert 30 inches to meters. Use 1 inch = 2.54 cm.

____ =

Ex. 2 Convert 4 lbs to grams. Use 1 oz = 28.35 grams.

____ =

Rates can be used as a **conversion factor** when doing unit conversions.

Example: 45 miles per hour.

________________= ______________

Ex. 1 How far can a person drive in 200 minutes if they are driving 45 miles per hour?

____ =

Ex. 2 A student can read 22 pages per hour. How many minutes will it take for a student to finish 240 pages of their summer reading?

____ =

Ex. 3 A student can type 38 words per minute. How many days will it take for a student to finish typing 50000 words for their Senior Project?

____ =

Adding and Subtracting Polynomial Expressions

VOCABULARY

<u>Polynomial</u> - An expression of algebraic terms, especially the sum of several terms that contain different _____________ of the same ___________________. (Ex: $5x^3 - 2x^2 + 7$)

For all addition and subtraction problems, we use CLT!

(___________________________ ____________________ ____________________)

Examples of like terms

$3x^2$, _____ 12xy, _____ 5√2, _____ $x^2\sqrt{x}$, _____

When combining like terms, we add or subtract ___________________________________!

Ex. 1 $5x + 7x^2 - 3x + 4$

Ex. 2 $6x^2 - 4x - 3x + 2 - 6x^2$

Ex. 3 $(5x^2 + 4x) + (3 - 7x)$

Ex. 4 $(4x^3 - 2x^2 + 5) + (3x^3 - 8x^2 - 3)$

When subtracting polynomials, we apply the subtraction to all parts of the polynomial behind the subtraction sign. Then, we CLT!

Ex. 1 $(5x + 7x^2) - (3x + 4)$

Ex. 2 $(6x^2 - 4x) - (-5x + 2 - 3x^2)$

Mixed practice!

A) $(3x^2 - 4x + 2) + (2x - 5x^2 + 6)$

B) $(2x^3 + 5x - 2) - (2x - 3x^3 - 2)$

C) $(-2x^2 + 7x - 12) - (20 - 4x^2)$

D) $(8x^3 - 4x) + (3x^2 - 9x + 7)$

Multiplying Polynomial Expressions

VOCABULARY

Monomial - A polynomial expression with ________ term. Example: $5x^2y$
Binomial - A polynomial expression with ________ terms. Example: $3x - 2y$
Trinomial - A polynomial expression with ________ terms. Example: $9x^2 + x - 1$

When multiplying monomials, multiply the __________________.
(Variable) · (Variable) changes the ____________________. Example: $x \cdot x =$ ______

Example: $5x^3 \cdot 7x =$ ________

Ex. 2 $-7x \cdot 11x =$ _______

Ex. 3 $9x \cdot -4 =$ ______

Ex. 4 $-6 \cdot -12x =$ _______

Ex. 5 $10xy \cdot 8xy =$ ______

===

For all multiplication problems which include __________________________, we use the _______________________________ ____________________________.

Example: $5x(2x - 3) =$ ____________

Ex. 2 $-3x(10x + 6) =$ ____________

Ex. 4 $7x(3x + -4) =$ ____________

Ex. 3 $-6x(-2x - 4) =$ ____________

Ex. 5 $5xy(4x - y) =$ ____________

===

For multiplying a binomial by a binomial,
distribute _____________________. Then, ________!

Example: $(2x + 3)(4x - 1)$

Distributive Property	Concrete Model

Problem	Distributive Property	Concrete Model
Ex. 2 $(3x + 6)(2x + 5)$		
Ex. 3 $(x - 8)(4x + 7)$		
Ex. 4 $(3x^2 - 7)(2x - 7)$		
Ex. 5 $(3x + 4)(3x - 4)$		
Ex. 6 $(2x - 6)^2$		

Area and Perimeter Applications

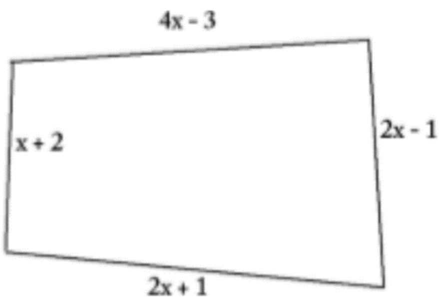
To find perimeter, ____.

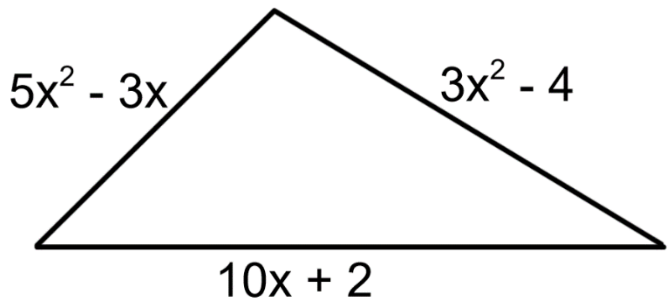
To find area, ____.

Area of Rectangle: **A = (base)(height)**

Area of Triangle: **A = ½(base)(height)**

Find an expression for the perimeters of the figures





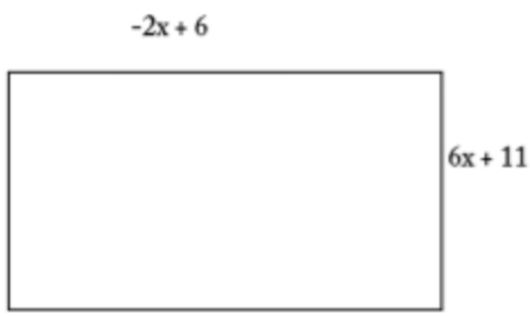
Rule for Perimeter =

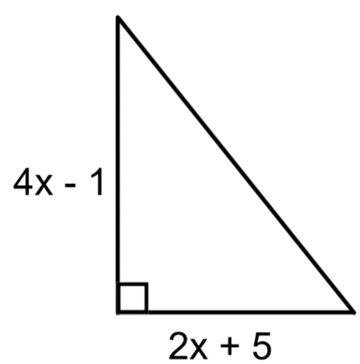
If x = 3, perimeter =

Rule for Perimeter =

If x = 2, perimeter =

Find an expression for the areas of the figures





Rule for Area =

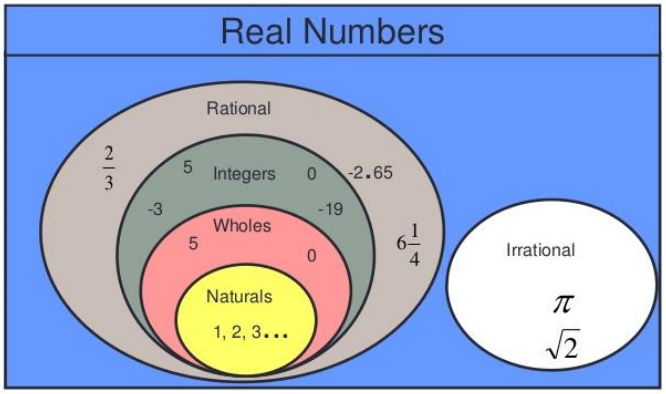
If x = 1, area =

Rule for Area =

If x = 4, area =

Rational and Irrational Numbers

Number Classifications
(from most general to most specific)



I. Real Numbers: a value that represents a quantity along a number line.

A. ***Rational Numbers:*** Numbers that can be expressed as a/b where a and b are integers.
Look like whole numbers, terminating decimals, or repeating decimals.

1. ***Integers:*** positive and negative whole numbers and zero.
   a. ***Whole Numbers:*** positive integers AND ZERO.
      i. ***Natural Numbers:*** positive integers. Does not include zero.

B. ***Irrational Numbers:*** Numbers that CANNOT be expressed as a/b where a and b are integers.
Look like non-terminating, non-repeating decimals.

II. Imaginary Numbers: a value that cannot be represented along a number line. Created by taking an even-root of a negative number like $\sqrt{-2}$. (An Algebra II topic!)

Example	Decimal Equivalence	Rational or Irrational?	Specific Type
1) 4.57			
2) -5/3			
3) $\sqrt{8}$			
4) $-\sqrt{9}$			
5) 12			
6) 12/5			
7) π			
8) $5\sqrt{81}$			
9) -4/7			
10) $2\sqrt{24}$			
11) 0			
12) $\frac{\sqrt{3}}{2}$			

Simplifying Radical Expressions

VOCABULARY

$$\sqrt[4]{3}$$

Product property: $\sqrt{ab}$ = ____________________

Example: $\sqrt{54}$ = _________________ = ________

Simplifying Radicals (Square-Roots):

Step 1: Factor the radicand into its prime factors by using a factor tree.

Step 2: Group same factors in groups of 2.

Step 3: For every group of 2 you have, you have a perfect square. Multiply your pairs back together into one radical and the leftovers into a second radical.

Step 4: Simplify.

Ex. A $\sqrt{24}$

Ex. B $\sqrt{27}$

Ex. C $\sqrt{225}$

Ex. D $\sqrt{x^5}$

Ex. E $\sqrt{108x^5y^4}$

Ex. F $3x\sqrt{18x^4}$

Multiplying Radical Expressions

<u>Multiplying Radicals (Square-Roots):</u>

Step 1: Factor radicands.

Step 2: Multiply coefficients and combine factors of radicands under one radical.

Step 3: Simplify radical like a single radical expression.

Ex. A $\sqrt{18} \cdot \sqrt{24}$

Ex. B $\sqrt{32x^3y} \cdot \sqrt{72xy^2}$

Ex. C $2x\sqrt{15x^2} \cdot 3\sqrt{20x^3}$

How would you describe the product of a rational and a rational number?

How would you describe the product of a rational and an irrational number?

How would you describe the product of an irrational and an irrational number?

Adding and Subtracting Radical Expressions

For all addition and subtraction problems, we use CLT!
(__________________________ ___________________ ____________________)

Examples of like terms

$3x^2$, _____ 12xy, _____ $5\sqrt{2}$, _____ $x^2\sqrt{x}$, _____

When combining like terms, we add or subtract ________________________________!

Ex. 1 $3\sqrt{2} - 2\sqrt{3} + 5\sqrt{2}$

Ex. 2 $5\sqrt{6} + 3\sqrt{6} - 7\sqrt{2} + 9\sqrt{2}$

When adding and subtracting radicals, simplify each radical before combining.

Ex. 1 $7\sqrt{96} + 5\sqrt{32}$

Ex. 2 $-\sqrt{18} - \sqrt{50} + \sqrt{2}$

Ex. 3 $3\sqrt{20} + 5\sqrt{45} - 7\sqrt{5}$

How would you describe the sum of a rational and a rational number?

How would you describe the sum of a rational and an irrational number?

How would you describe the sum of an irrational and an irrational number?