8. Techniques of Differentiation - Classwork

You might wonder why this concept of a derivative, a slope of a tangent line to a curve, is important enough to have an entire math course based on it. When you examine applications of the derivative later on in the course, you will understand its significance. But for now, trust that derivatives are important and we need to learn how to take them rather than go through the cumbersome limit process. We will spend some time learning how to take derivatives of all sorts of functions in order that when we learn applications, we will be able to use real-life situations. For the different rules below, for each problem, take the derivative using correct notation.

1) **The constant rule**: The derivative of a constant is 0. That is, if c is a real number, then $\frac{d}{dx}[c]=0$. From a geometric point of view, the graph of $y=c$ is a horizontal line and at any point along the line, its slope is 0.

a) $y=9$ $y'=0$

b) $f(x)=0$ $f'(x)=0$

c) $s(t)=-8$ $s'(t)=0$

d) $y=\frac{1}{\pi^3}$ $y'=0$

2) **The single variable rule**: The derivative of $x=1$: $\frac{d}{dx}[x]=1$. This is consistent with the fact that the slope of the line $y=x$ is 1 at any point on the line.

a) $y=x$ $y'=1$

b) $f(x)=x$ $f'(x)=1$

c) $s(t)=t$ $s'(t)=1$

d) $g(T)=T$ $g'(T)=1$

Functions for which the derivative exists are called **differentiable**. A function may be differentiable at some x-values and not differentiable at others.

3) **The Power Rule:** If n is a rational number, then the function $f(x)=x^n$ is differentiable and $\frac{d}{dx}[x^n]=nx^{n-1}$. Usually, some work will need to be done to get the function in the proper form so the power rule can be used.

Determine the derivatives and determine any values of x for which the function is differentiable.

a) $y=x^2$ $\boxed{y'=2x}$

b) $f(x)=x^6$ $\boxed{f'(x)=6x^5}$

c) $s(t)=t^{30}$ $\boxed{s'(t)=30t^{29}}$

d) $y=\sqrt{x}$ $y=x^{\frac{1}{2}}$ $y'=\frac{1}{2}x^{-\frac{1}{2}}$

In general, we prefer to leave answers with positive exponents.

e) $y=\frac{1}{x}$ $y=x^{-1}$ $\boxed{y'=-x^{-2}}$ $=-\frac{1}{x^2}$

f) $f(x)=\frac{1}{x^3}$ $f(x)=x^{-3}$ $\boxed{f'(x)=-3x^{-4}}$

g) $s(t)=\frac{1}{\sqrt[3]{t}}$ $s(t)=t^{-1/3}$ $\boxed{s'(t)=-\frac{1}{3}t^{-\frac{4}{3}}}$

h) $y=\frac{1}{x^{3/4}}$ $y=x^{-3/4}$ $\boxed{y'=-\frac{3}{4}x^{-\frac{7}{4}}}$

4) **The constant multiple rule**: If f is a differentiable function and c is a real number, $\frac{d}{dx}[c \cdot f(x)] = cf'(x)$.

Take the derivative of each function. Use proper notation.

a) $y = \frac{2}{x^2}$ $\boxed{y' = -4x^{-3}}$

b) $f(x) = \frac{4x^3}{3}$ $\boxed{f'(x) = 4x^2}$

c) $s(t) = -t^5$ $\boxed{s'(t) = -5t^4}$

d) $y = 4\sqrt{x}$ $y = 4x^{\frac{1}{2}}$ $\boxed{y' = 2x^{-\frac{1}{2}}}$

e) $y = \frac{5}{3x^3}$ x^{-3} $\boxed{y' = 5x^{-4}}$ $\cancel{3}\ \frac{5}{\cancel{3}}x^{-3}$

f) $f(x) = \frac{-5}{(3x)^3} = \frac{-5}{27x^3}$ $-\cancel{3} \cdot \frac{-5}{\cancel{27}}x^{-3}$ $\boxed{f'(x) = \frac{5}{9}x^{-4}}$

g) $s(t) = \frac{40}{\sqrt{t}}$ $40t^{-\frac{1}{2}}$ $\boxed{s'(t) = -20t^{-\frac{3}{2}}}$

h) $y = \frac{12}{\sqrt[3]{x^5}}$ $12x^{-5/3 \cdot \frac{3}{3}}$ $-\frac{5}{\cancel{3}} \cdot \cancel{12}^{4}$ $\boxed{y' = -20x^{-8/3}}$

5) **The sum or difference rule**. The derivative of a sum or difference is the sum or difference of the derivatives.

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

Take the derivative of each function. Use proper notation.

a) $y = x^2 + 5x - 3$ $\boxed{y' = 2x + 5}$

b) $f(x) = x^4 - \frac{3}{2}x^3 + 2x^2 + x - 6$ $\boxed{f'(x) = 4x^3 - \frac{9}{2}x^2 + 4x + 1}$

c) $y = (2x-3)^2$ $(2x-3)(2x-3)$ $4x^2 - 12x + 9$ $\boxed{y' = 8x - 12}$

d) $y = \frac{4}{x} - \frac{4}{x^2} + \frac{4}{x^3}$ $\boxed{y' = -4x^{-2} + 8x^{-3} - 12x^{-4}}$

e) $f(x) = 6\sqrt{x}(2\sqrt{x} - 3)$ $\boxed{f'(x) = 12 - 9x^{-\frac{1}{2}}}$

f) $y = \frac{(x^2 - x + 1)^2}{2}$ $\boxed{y' = 2x^3 - 3x^2 + 3x - 1}$

g) $y = \frac{8}{\sqrt{x}} - \frac{6}{\sqrt[3]{x}}$ $\boxed{y' = -4x^{-3/2} + 2x^{-4/3}}$

h) $y = \frac{9x - 3\sqrt{x}}{x}$ $\boxed{y' = \frac{3}{2}x^{-3/2}}$

i) $y = \frac{x^2 - 6x - 16}{2x + 4}$ $y' = \frac{1}{2}$

9.

$$y = \frac{8}{\sqrt{x}} - \frac{6}{\sqrt[3]{x}}$$

$$= \frac{8}{x^{\frac{1}{2}}} - \frac{6}{x^{\frac{1}{3}}} = 8x^{-\frac{1}{2}} - 6x^{-\frac{1}{3}}$$

h.

$$y = \frac{9x - 3\sqrt{x}}{x}$$

$x^{\frac{1}{2}}$

$\frac{1}{2} - 1$

$$= \frac{9\cancel{x}}{\cancel{x}} - \frac{3\sqrt{x}}{x^1} \qquad \frac{3x^{\frac{1}{2}}}{x^1}$$

$$= 9 - 3x^{-\frac{1}{2}}$$

e. $6\sqrt{x}\,(2\sqrt{x}-3)$

$12x-18\sqrt{x}$

$12x-18x^{\frac{1}{2}}$

8. Techniques of Differentiation - Homework

For the following functions, find $f'(x)$ and $f'(c)$ at the indicated value of c. Use proper notation.

1. $f(x)=x^2-6x+1,\ c=0$

$f'(x)=2x-6$

$f'(0)=-6$

2. $f(x)=\frac{1}{x}-\frac{3}{x^2}+\frac{4}{x^3},\ c=-1$

$f'(x)=-x^{-2}+6x^{-3}-12x^{-4}$

$f'(-1)=-19$

3. $f(x)=3\sqrt{x}-\frac{1}{\sqrt[3]{x}},\ c=1$

$f'(x)=\frac{3}{2}x^{-\frac{1}{2}}+\frac{1}{3}x^{-4/3}$

$f'(1)=11/6$

For the following functions, find the derivative using the power rule. Use proper notation.

4. $y=\pi^5$

$\boxed{y'=0}$

5. $y=\frac{x^2+2}{6}$

$\frac{1}{6}x^2+1/3$

$\boxed{y'=\frac{1}{3}x}$

6. $y=\frac{1}{a}\left(x^2-\frac{1}{b^2}x+c\right)$, a,b,c const

$\frac{1x^2}{6}+\frac{2}{6}$

$y'=\frac{2}{a}x-\frac{1}{ab^2}$

7. $y=\frac{8}{3x^2}$

$\boxed{y'=-\frac{16}{3}x^{-3}}$

8. $y=\frac{-9}{(3x^2)^3}$

$\frac{-9}{27x^6}=\frac{-1}{3}x^{-6}$

$\boxed{y'=2x^{-7}}$

9. $y=\frac{6x^{3/2}}{x}$

$\boxed{y'=3x^{-1/2}}$

10. $y=\frac{4x^2-5x+6}{3}$

$\boxed{y'=\frac{8}{3}x-5/3}$

11. $y=\frac{x^2-6x+2}{2x}$

$\boxed{y'=\frac{1}{2}-x^{-2}}$

12. $y=\frac{x^3+8}{x+2}$

$\frac{\cancel{(x+2)}(x^2-2x+4)}{\cancel{x+2}}$

$\boxed{y'=2x-2}$

13. $y=x^4-\frac{3}{2}x^3+5x^2-6x-2$

$\boxed{y'=4x^3-\frac{9}{2}x^2+10x-6}$

14. $y=\frac{x^3-3x^2+10x-5}{x^2}$

$\boxed{y'=1-10x^{-2}+10x^{-3}}$

15. $y=(x^2+4x)(2x-1)$

$\boxed{y'=6x^2+14x-4}$