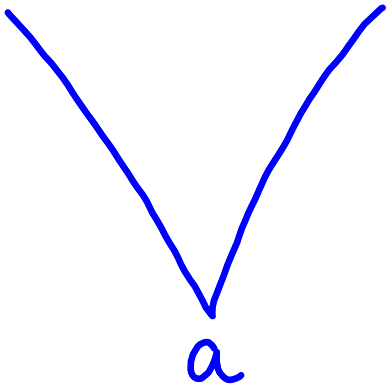
Conclusion about differentiablity:

Most functions in calculus are differentiable:

polynomial ·
rational ·
trigonometric ·
exponential ·
logarithmic ·
compositions of the above



If a function has a derivative at $x = a$, then it is continuous at $x = a$.

The converse IS NOT TRUE!

2.2 Basic Differentiation Rules

RULE 1 The Constant Rule

If f is the function with the constant value c, then

$$\frac{df}{dx} = \frac{d}{dx}(c) = 0.$$

$y = 5$
$y' = 0$

$f(x) = 5$
$f'(x) = 0$

RULE 2 The Power Rule

If n is a rational number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}.$$

$x^4 = 4x^{4-1} = 4x^3$

RULE 3 The Constant Multiple Rule

If u is a differentiable function of x and c is a constant, then

$$\frac{d}{dx}(cu) = c\frac{du}{dx}.$$

$f(x) = 3x^2$

$2 \cdot 3x^1 = 6x$

$\boxed{3 \cdot f(x) = x^2}$

$3 \cdot \boxed{2x^1} = 6x$

RULE 4 The Sum and Difference Rule

If u and v are differentiable functions of x, then their sum and difference are differentiable at every point where u and v are differentiable. At such points,

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}.$$

$f(x) = x^2 + 5x + \frac{1}{x}$

$f(x) = x^{2-1} + 5x^{1-1} + x^{-1-1}$

$\boxed{f'(x) = 2x + 5 - 1x^{-2}}$

Example

Find $\frac{dp}{dt}$ if $p = t^{3-1} + 6t^{2-1} - \frac{5}{3}t^{1} + 16.$

$\boxed{p' = 3t^2 + 12t - \frac{5}{3}}$

$y = \underline{5x}\ y' = 5$

Example Find $f'(x)$.

$$f(x) = \frac{1}{x^2} \qquad f(x) = x^{-2}$$

$$f'(x) = -2x^{-3} = \frac{-2}{x^3}$$

Example Find $\frac{dy}{dx}$.

$$y = \sqrt{x} - 4 \qquad y = x^{\frac{1}{2}} - \frac{2}{2} - 4$$

$$y' = \frac{1}{2}x^{-\frac{1}{2}}$$

Examples Find the derivative of each function.

1. $f(x) = 3\sqrt{x} + 8x = 3x^{\frac{1}{2}} + 8x$

$f'(x) = \frac{3}{2}x^{-\frac{1}{2}} + 8$

2. $f(x) = \dfrac{x + 3\sqrt{x}}{x^2} = \dfrac{x^1}{x^2} + \dfrac{3x^{\frac{1}{2}}}{x^2}$ $\quad \frac{1}{2} - 2$

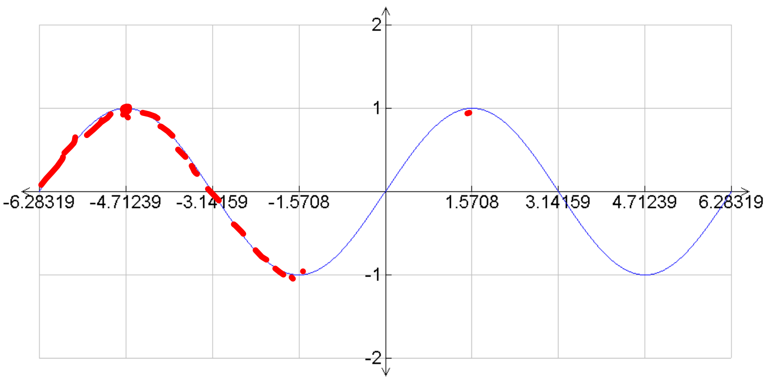
$f'(x) = -x^{-2} - \frac{9}{2}x^{-\frac{5}{2}} = x^{-1} + 3x^{-\frac{3}{2}-\frac{2}{2}}$

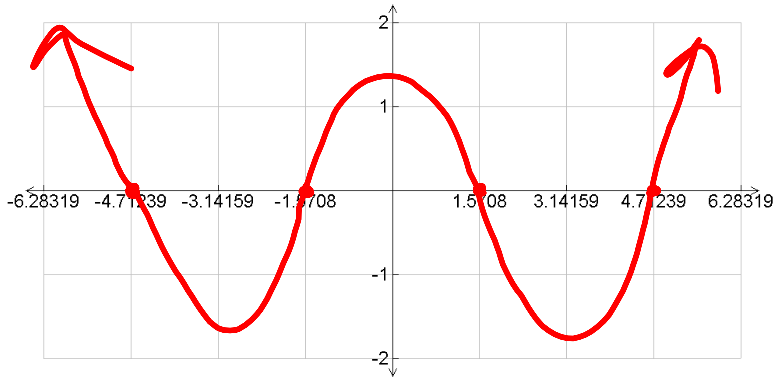
3. $f(x) = \dfrac{5}{x^{2/3}} = 5x^{-2/3 - \frac{3}{3}}$

$f'(x) = \frac{-10}{3}x^{-\frac{5}{3}}$

What is the derivative of sin*x*?

y = sin *x*





The Derivatives of Sine and Cosine

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

Memorize These!

More Trig Derivatives

$$\frac{d}{dx}\tan x = \sec^2 x, \qquad \frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\cot x = -\csc^2 x, \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

<u>Example</u> Find the derivative.

$$f(x) = \left(\frac{\pi}{2}\sin x\right)\left(-\cos x\right)$$

$$f'(x) = \frac{\pi}{2}\cos x - (-\sin x)$$

$$f'(x) = \frac{\pi}{2}\cos x + \sin x$$

<u>Horizontal Tangents</u>

<u>Example</u> Determine the point(s) at which the function has a horizontal tangent. $m = 0$

$y = 3x^4 - 11x^2 - 4$

$y' = 12x^3 - 22x = 0$

$= 2x(6x^2 - 11) = 0$

$\frac{2x}{2} = \frac{0}{2}$ $\quad$ $6x^2 - 11 = 0$

$x = 0$ $\quad$ $+11 \quad +11$

$\frac{6x^2}{6} = \frac{11}{6}$

$\sqrt{x^2} = \pm\sqrt{\frac{11}{6}}$

$x = \pm\sqrt{11/6}$