5.6 Integration by Parts

$\frac{d}{dx}[f(x)g(x)] = f'g + g'f$

So,

$\int [\boxed{f(x)g'(x)} + g(x)f'(x)]\,dx = f(x)g(x)$

and this equation becomes

$\int f(x)g'(x)\,dx = f(x)g(x) - \underline{\int g(x)f'(x)\,dx}$

Pg. 11-12

1. $\frac{1}{2}\int_1^4 u^3\,du = \boxed{31.875u^2}$

$\frac{1}{2}\cdot\frac{1}{4}u^4\Big]_1^4 \quad \frac{1}{8}u^4\Big]_1^4 =$

$32 - \frac{1}{8} = 31.875$

2. $-\frac{1}{2}\int_9^0 u^{\frac{1}{2}}\,du$

$-\frac{1}{2}\cdot\frac{2}{3}u^{\frac{3}{2}}\Big]_9^0 = 9.0u^2$

4. $\frac{1}{2}\int_1^9 u^{-\frac{1}{2}}\,du$

$\frac{1}{2}\cdot 2\cdot u^{\frac{1}{2}}\Big]_1^9 = 2.0u^2$

5. $\frac{1}{2}\int_1^6 u^{-2}\,du$

$\frac{1}{2}\cdot -1 \cdot u^{-1}\Big]_1^6 =$ u^2

$-\frac{1}{2}u^{-1}\Big]_1^6 =$

$-\frac{1}{2}\cdot\frac{1}{6}$

$-\frac{1}{12} - \left(-\frac{1}{2}\right) = .4167\ u^2$

7. $-\int_1^0 u^3\,du$ $u=\cos x$ $\frac{du}{dx} = -\sin x$

$-\frac{1}{4}u^4\Big]_1^0 = .25u^2$

$0 - \left(-\frac{1}{4}\right)$

8. $\int_0^1 u^{\frac{1}{2}}\,du$

$\frac{2}{3}u^{\frac{3}{2}}\Big]_0^1 = \frac{2}{3}u^2$

10. $2\int_{\pi/2}^{\pi} \sin u\,du$

$2\cos u\Big]_{\pi/2}^{\pi}$

$-2 - 0 = -2\,u^2$

In order to remember this equation,
let u = f(x) and dv = g'(x).

$$\boxed{\int u\,dv = uv - \int v\,du}$$

* Choose "u" so that "du" is simpler than "u".

* Choose "dv" so that "v" is something that you can find.

Evaluate each integral.

1. $\int \underset{u}{x} \cdot \underbrace{\cos x\, dx}_{dv}$

$\boxed{\int u\,dv = uv - \int v\,du}$

$u = x \quad du = 1\,dx$

$v = \sin x \quad dv = \cos x\, dx$

$\int x \cos x\, dx = x \sin x - \int \sin x\, dx$

$u\,dv \quad = u \cdot v \quad - \int v\,du$

$= x\sin x - (-\cos x) + C$

$\boxed{x \cdot \sin x + \cos x + C}$

2. $\int x e^x\, dx$

$\boxed{\int u\,dv = uv - \int v\,du}$

$u = x \quad du = dx$

$v = e^x \quad dv = e^x\,dx$

$= x e^x - \int e^x\, dx$

$= \boxed{xe^x - e^x + C}$

4. $\int \sqrt{x} \ln x \, dx$

$u = \ln x \quad du = \frac{1}{x}dx$

$\int u\,dv = uv - \int v\,du$

$v = \frac{2}{3}x^{3/2} \quad dv = \sqrt{x}\,dx$

$x^{\frac{1}{2}}$

$= \frac{2}{3}x^{3/2}\ln x - \int \frac{\frac{2}{3}x^{3/2}}{x}\,dx$

$\frac{2}{3} \cdot \int \frac{2}{3}x^{\frac{1}{2}+\frac{2}{3}}\,dx$

$= \frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{4}{9}x^{\frac{3}{2}} + C$