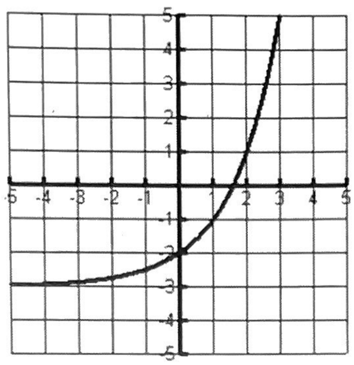
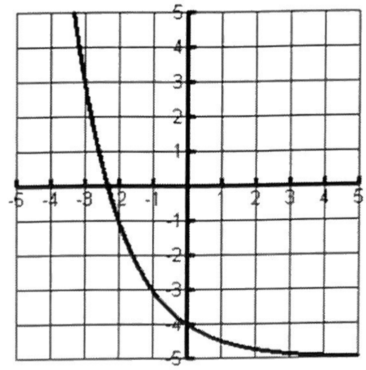
I. Exponential Characteristics





1. D: $-\infty < x < \infty$ R: $-3 < y < \infty$

Extrema: none

B $>$ 1 Exponential Growth

X-Int: (1.5,0) Y-Int: (0,-2)

Asymptote: $y=-3$ Inc OR Dec? (Circle)

End Behavior: As x → $-\infty$, y → -3
As x → ∞, y → ∞

Average rate of change for -1 < x < 1

(-1,-2.5)
(1,-1)

$$\frac{(-1)-(-2.5)}{(1)-(-1)} = \frac{3}{4}$$

4. D: $-\infty < x < \infty$ R: $-5 < y < \infty$

Extrema: none

B $<$ 1 Exponential Decay

X-Int: (-2.3,0) Y-Int: (0,-4)

Asymptote: $y=-5$ Inc OR Dec? (Circle)

End Behavior: As x → $-\infty$, y → ∞
As x → ∞, y → -5

Average rate of change for -2 < x < -1

(-2,-1)
(-1,-3)

$$\frac{(-3)-(-1)}{(-1)-(-2)} = -2$$

II. Exponential Transformations
Describe the parent, growth or decay, & transformations for the following functions

	Parent Function	Growth or Decay	List Transformations
$f(x) = -2^{x-4} + 3$	2^x	Growth	· Reflect over x-axis · shift right 4 · shift up 3
$g(x) = \frac{2}{3}(4)^{x+5}$	4^x	Growth	· compress by 2/3 · shift left 5
$h(x) = -\frac{5}{2}\left(\frac{1}{3}\right)^{x-1} + 7$	$\frac{1}{3}^x$	Decay	· reflect over x-axis · stretch by 5/2 · shift right 1 and up 7

Write the new function as g(x) using the parent function $y = \frac{1}{3}^{x}$ and the transformations.

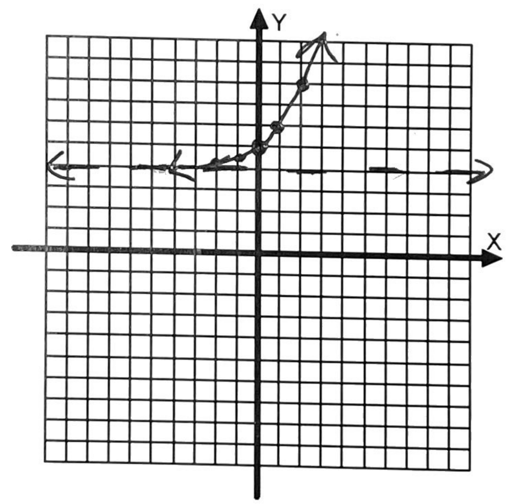
1. Shift right 3 and down 6 — $g(x) = \frac{1}{3}^{x-3} - 6$
2. Reflect over x-axis, shift up 4, and stretch by 2 — $g(x) = -2\left(\frac{1}{3}\right)^{x} + 4$
3. Reflect over x-axis, shift right 9, and compress by 1/5 — $g(x) = -\frac{1}{5}\left(\frac{1}{3}\right)^{x-9}$

III. Exponential Graphs

Graph the following functions carefully by filling out the table and graphing the coordinate pairs. Remember to shift your table by the horizontal shift.

A. Graph $y = 2^x + 4$

X	Y
-2	4.25
-1	4.5
0	5
1	6
2	8



Domain:

$-\infty < x < \infty$

Range:

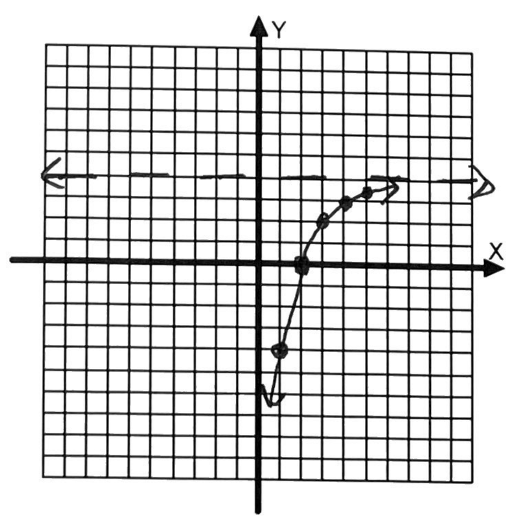
$4 < y < \infty$

Asymptote:

$y = 4$

B. Graph $y = -2\left(\frac{1}{2}\right)^{x-3} + 4$

X	Y
1	-4
2	0
3	2
4	3
5	3.5



Domain:

$-\infty < x < \infty$

Range:

$-\infty < y < 4$

Asymptote:

$y = 4$

IV. Exponential Equations

1. A population of 500 elk is released in a wildlife preserve. Every 3 years, the population grows by 16.4%.
   a. Write an exponential equation that represents the number of elk x years after the release.

   $1 + .164 = 1.164 \quad f(x) = 500(1.164)^{x/3}$

   b. After 5 years, how many elk are there?

   $f(5) = 500(1.164)^{5/3} = 644.01 = 644$ elk

2. A store receives a shipment of 1000 greeting cards. Every 2 days, the store sells 2.5% of its stock of cards. Let x = # of days passed; f (x) = # of cards remaining in the store.
   a. Write an exponential equation that relates x and f (x), using the given information.

   $1 - .025 = .975 \quad f(x) = 1000(0.975)^{x/2}$

   b. After 15 days, how many cards will remain in the store?

   $f(15) = 1000(0.975)^{15/2} = 827.06 = 827$ cards

3. Smily Suzie is an LHS student who likes making new friends. Before the start of the school year, she has 20 friends. Each day of school, the number of friends that she has increases by 11%.
   a. Write an exponential equation that represents how many friends Suzie has after x days.

   $1 + .11 = 1.11 \quad f(x) = 20(1.11)^x$

   b. There were 18 school days in September. How many friends did Suzie have at the end of September?

   $f(18) = 20(1.11)^{18} = 130.87 = 131$ friends

4. Suppose the population of a nation is growing by 9% per decade. If the population was 30,000,000 in 1975, what will the population be in 2019, to the nearest million?

   $2019 - 1975 = 44 \quad f(44) = 30{,}000{,}000(1.09)^{44/10}$

   $1 + .09 = 1.09 \quad = 43{,}832{,}661.91 \approx 44$ million people

5. According to legend, in 1626 Manhattan Island was purchased for trinkets worth about $24. If the $24 had been invested at a rate of 6% interest per year, what would be its value in 2006? Compare this with a total of $802.4 billion in assessed values for Manhattan in 2006.

   $f(380) = 24(1.06)^{380}$

   $1 + .06 = 1.06$

   $2006 - 1626 = 380 \quad = 9.92\text{E}10 = 9.92 \times 10^{10} = 99{,}200{,}000{,}000$

   $\approx \$99$ billion

6. If the price of theater tickets increases by 2% 3 times per year, how much will a $100 ticket cost 5 years from now?

$1+.02=1.02$ $\qquad f(5)=100(1.02)^{3(5)}=\$134.59$

7. You bought a new car for $28,000. If the car depreciates (loses value) at a rate of 15% per year, how much will it be worth in 5 years?

$1-.15=0.85$ $\qquad f(5)=28000(0.85)^5=\$12423.75$

8. You charged $200 to your credit card. Your credit card charges you 8.9% per month. Assume the card doesn't charge you late fees.
   a. If you don't make a payment, how much will you owe after 5 months? How much more than your original charge did you pay in interest?

$1+.089=1.089$ $\qquad f(5)=200(1.089)^5=\$306.32$

Interest $=306.32-200=\$106.32$

   b. If you don't make a payment, how much will you owe after 2 years? How much more than your original charge did you pay in interest?

$f(24)=200(1.089)^{24}=\$1547.74$ $\qquad$ Interest $=1547.74-200=\$1347.74$

Use the compound interest formula for the following problems. $A=P\left(1+\frac{r}{n}\right)^{nt}$

9. You are saving $3000 at 5% compounded monthly.
   a. How much do you have after 5 years?

$$A=3000\left(1+\frac{.05}{12}\right)^{12(5)}=\$3850.08$$

   b. How much do you have after 10 years?

$$A=3000\left(1+\frac{.05}{12}\right)^{12(10)}=\$4941.03$$

   c. How much do you have after 20 years?

$$A=3000\left(1+\frac{.05}{12}\right)^{12(20)}=\$8137.92$$

10. You are saving $4500 at 4% compounded daily.
   a. How much do you have after 5 years?

$$A=4500\left(1+\frac{.04}{365}\right)^{365(5)}=\$5496.25$$

   b. How much do you have after 10 years?

$$A=4500\left(1+\frac{.04}{365}\right)^{365(10)}=\$6713.06$$

   c. How much do you have after 20 years?

$$A=4500\left(1+\frac{.04}{365}\right)^{365(20)}=\$10014.50$$

V. Exponential Equations from Tables

Your car loan is growing by the following amount.

Month	Amount Owed
1	$14000
2	$14280
3	$14565.6
4	$14856.91
5	$15154.05

A. Write an equation to represent how much money you owe on your car after x months

$r = \frac{14280}{14000} = 1.02 \quad f(x) = 14000(1.02)^x$

B. How much money did you originally borrow to buy your car?

$14000

C. What interest rate is the bank charging?

$1.02 - 1 = 0.02 \therefore$ 2% interest per month

D. How much money will you owe after 2 years?

$f(24) = 14000(1.02)^{24} = \$22518.12$

E. How much interest will you pay after 2 years?

$22518.12 - 14000 = \$8518.12$

VI. Geometric Sequences

Write an explicit rule for the following sequences. Use your rule to find the 8th term.

A. 4, 12, 36, 108, ...

$r = \frac{12}{4} = 3$

$a_n = 4(3)^{n-1}; \ a_8 = 8748$

B. 54, 36, 24, 16

$r = \frac{36}{54} = \frac{2}{3}$

$a_n = 54\left(\frac{2}{3}\right)^{n-1}; \ a_8 = \frac{256}{81}$

C. 3, -12, 48, -192

$r = -\frac{12}{3} = -4$

$a_n = 3(-4)^{n-1}$

$a_8 = -49152$

Write a recursive rule for the following sequences.

A. 4, 12, 36, 108, ...

$a_1 = 4$

$a_n = 3 \cdot a_{n-1}$

B. 54, 36, 24, 16

$a_1 = 54$

$a_n = \frac{2}{3} \cdot a_{n-1}$

C. 3, -12, 48, -192

$a_1 = 3$

$a_n = -4 \cdot a_{n-1}$

Use the following rules to write the first 4 terms of the sequence.

A. $a_n = -2a_{n-1}; \ a_1 = 5$

5, -10, 20, -40

B. $a_n = 5(-4)^{n-1}$

5, -20, 80, -320

C. $a_n = 100\left(\frac{1}{2}\right)^{n-1}$

100, 50, 25, 12.5

D. $a_n = \frac{4}{3}a_{n-1}; \ a_1 = 27$

27, 36, 48, 64