key

Calculus Review

1. Find the following limits.

a. $\lim_{x \to -1} \frac{x-3}{x^2+4x+2}$ 4

b. $\lim_{x \to -3} 5x+2$ −13

c. $\lim_{x \to -3} \frac{2x^2+5x-3}{x^2-x-12}$ 1

d. $\lim_{x \to -1} \frac{1}{x-1}$ −1/2

e. $\lim_{x \to 0} \frac{x\tan x - \tan x}{x}$ −1

f. $\lim_{x \to \pm\infty} \frac{3x^2-4x+1}{2x^2+5}$ 3/2

g. $\lim_{x \to 0} \frac{\sin x}{3x^2+2x}$ 0.5

h. $\lim_{x \to 2^+} \frac{1}{x^2-4}$ ∞

i. $\lim_{x \to \infty} \frac{4x^2-12x}{2x-6}$ ∞

j. $\lim_{x \to 2^-} \frac{1}{x^2-4}$ $-\infty$

2. $f(x) = \begin{cases} x-2 & x<1 \\ 1 & x=1 \\ 4-x & x>1 \end{cases}$ What is $\lim_{x \to 1^+} f(x)$ and $\lim_{x \to 1^-} f(x)$?

3 −1

3. Find the limits of each of the following as $x \to \infty$:

a. $f(x) = \frac{5x^3-4x^2}{4x^3-7}$ $\frac{5}{4}$

b. $g(x) = \frac{5x^3-4x^2}{4x^4-7x}$ 0

c. $k(x) = \frac{7x^2+3}{x-4}$ ∞

4. Find the points at which $y = \frac{1}{x^2-3x-4}$ is not continuous. 4, −1

5. Define $g(-2)$ so that $g(x) = \frac{x^3-4x}{x^2+5x+6}$ is continuous at $x=-2$. y = 8

7. Which of the following are continuous at $x=3$?

~~a.~~ $f(x) = \frac{x^2+4x+3}{x^2-9}$

(b.) $f(x) = \frac{|x+4|}{x+3}$

(c.) $f(x) = \frac{x^2}{x^2+10x+21}$

~~d.~~ $f(x) = \begin{cases} 3x+2 & x<0 \\ (x-1)^2 & 0 \le x < 3 \\ x-2 & x \ge 3 \end{cases}$

8. Which of these functions is/are defined at $x=-5$? For which of these functions does the limit $x \to -5$ exist? Which of these functions is/are continuous for $x=-5$?

a. $v(x) = \frac{x(x+5)(x-5)}{x+5}$ limit exists

b. ~~$m(x) = \frac{3x+5}{x+5}$~~

c. $t(x) = x^3 - 7$ defined ✓ @ x = −5, limit exists ✓, cont ✓

d. ~~$k(x) = \frac{\frac{1}{2}x}{x+5}$~~

e. $p(x) = \frac{x^2-5x-14}{x+3}$ defined ✓ @ x = −5, limit exists ✓, cont ✓

9. Find dy/dx for the following

a. $y = 5x^4 - 2x^3 + 3x^2 + 6$ b. $y = -\frac{x^3}{2} + 2x$ c. $y = (x - 3)(x^2 + 2)$

d. $y = \frac{3x + 2}{x - 4}$ e. $y = 3x^3 - \cot x$ f. $y = \frac{2x}{1 + \sec x}$

f. $y = \tan x^2$ g. $y = (\sin x)^2$ h. $y = (x^2 + 4)^{\frac{1}{2}}$

i. $y = \sin(\cos x)$ j. $y = \frac{2x^3 - 1}{x^2}$ k. $xy + y^2 = 1$

l. $x = \cos y$ m. $y^3 - x^3 = x^2$ n. $y = \ln(3x + 2)$

o. $y = x^3 \ln x$ p. $y = \ln(x^4 - 1)^3$ q. $y = e^{2X-3}$

r. $y = x^2 e^{-x}$ s. $y = e^{3x^3}$ t. $y = 5^{3x}$

u. $y = \log_4 5x$ v. $e^{2xy} + x^2 - y = 1$

see answers attached

10. The position of a particle is given by $p(t) = t^3 - 12t^2 + 45t$, where t is measured in seconds and p(t) is in meters.

a. When is the particle at rest?

when $v(t) = 0$

$v(t) = 3t^2 - 24t + 45$

$0 = 3t^2 - 24t + 45$

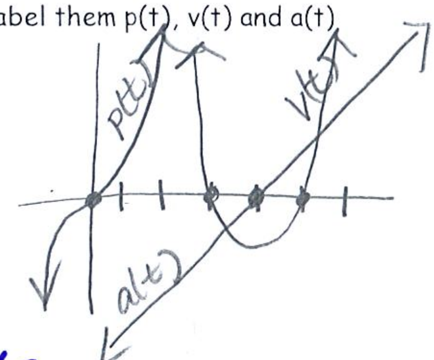
$0 = 3(t - 3)(t - 5)$

at $t = 3, 5$ seconds

b. Find the acceleration function. What is the acceleration at 3 seconds?

$a(t) = 6t - 24$ $a(3) = 6(3) - 24 = -6$ m/s^2

c. Graph the position, velocity and acceleration functions on the graph. Label them p(t), v(t) and a(t) respectively.



d. When is the particle speeding up? Use interval or inequality notation.

Speed up $(3,4) \cup (5,\infty)$

e. When is the particle slowing down? Use interval or inequality notation.

Slow down $(0,3) \cup (4,5)$

f. What is the velocity of the particle at t = 2 seconds?

$v(2) = 3(2)^2 - 24(2) + 45 = 50$ m/s

11. Find the second derivative: $y = 2x^2 + \sin 2x$

$\frac{dx^2}{dy} = 4 - 4\sin 2x$

12. Find the extrema (max/min), intervals of inc/dec, points of inflection and intervals of concave up/concave down.

a. $f(x) = 4x - x^2$

inc$(-\infty, 2)$ dec $(2, \infty)$

max @ 2 no pts of inf.

b. $y = \frac{x + 1}{x - 1}$

no max/min

change concavity @ $x = 1$

CD$(-\infty, 1)$ CU$(1, \infty)$

c. $y = 3xe^x$

min @ -1

pt of inf @ $x = 2$

CU$(-\infty, 2)$ dec$(-\infty, -1)$

CD$(2, \infty)$ inc $(-1, \infty)$

13. Find the following integrals

a. $\int \sqrt[4]{x^3} + 1\, dx$ $\frac{4}{7}x^{7/4} + x + C$

b. $\int (3 - x)x^3\, dx$ $\frac{3}{4}x^4 - \frac{1}{5}x^5 + C$

c. $\int 11\, dx$ $11x + C$

d. $\int 2x^2 - \cos x\, dx$

$\frac{2}{3}x^3 - \sin x + C$

e. $\int 2x(3x^2 + 1)^3\, dx$

$\frac{1}{12}(3x^2 + 1)^4 + C$

f. $\int \frac{\sin x}{\cos^3 x}\, dx$

$\frac{1}{2\cos^2 x} + C$

g. $\int_0^1 \frac{x^4}{\sqrt{x^5+9}}\,dx$ = .065

h. $\int \frac{1}{x-7}\,dx$ ln(x-7)+C

i. $\int \frac{x^3}{x^4-4}\,dx$ $\frac{1}{4}\ln(x^4-4)+C$

j. $\int_e^{e^3} \frac{3}{x(\ln x^3)}\,dx$ ln3

k. $\int -3x^2 e^{-x^3}\,dx$ $e^{-x^3}+C$

l. $\int \frac{e^{3x}}{e^{3x}+6}\,dx$ $\frac{1}{3}\ln(e^{3x}+6)+C$

16. Find the area between the two curves: $f(x) = -x^2 - 2x + 3$, $g(x) = -x - 3$

$\int_{-3}^{2} -x^2-2x+3-(-x-3)\,dx = \int_{-3}^{2} -x^2-x+6 = -\frac{1}{3}x^3-\frac{1}{2}x^2+6x\Big]_{-3}^{2} = 20.8\overline{3}$

17. Find the equation of the line tangent to the graph of $y = -3x^2 + 2x - 1$ at x=1.

$y' = -6x+2$ $y'(1) = -6(1)+2 = -4 = m$ @ $(1,-2)$ $y-(-2) = -4(x-1)$ $y+2 = -4x+4$ $y = -4x+2$

18. A 18-foot ladder is leaning against a building. It is sliding away from the wall at a rate of 3 feet per second. How fast is the ladder moving down the wall, when the base is 8 feet from the wall?

−1.49 ft/s

19. A spherical balloon is inflated with helium at the rate of $200\pi\ ft^3/\text{min}$. How fast is the balloon's radius increasing when the radius is 7 ft.?

1.02 ft/s

20. Determine if the Mean Value Theorem applies; if so find all values of c such that $f'(c) = \frac{f(b)-f(a)}{b-a}$: $f(x) = -x^2 + 2x$, $[2,4]$

x=3

21. Find two positive numbers such that the product is 432 and the sum of the first and three times the second is a minimum.

12, 36

22. You are to create an open top box out of a 10 x 12 ft sheet of scrap metal of maximum volume by cutting out congruent squares from each corner and folding up the sides. What is the **maximum volume**?

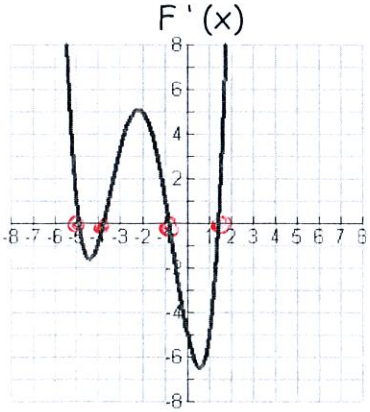
$V' = 12x^2 - 88x + 120 = 0$ Use calculator or quadratic

$x = 1.81$

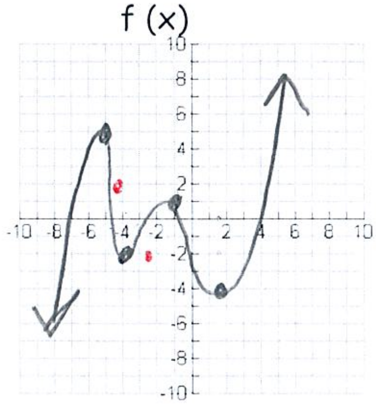
$V \approx 96\ ft^3$

23. Given the first derivative f ' (x) graph below, sketch the original function f (x) and the second derivative f "(x).

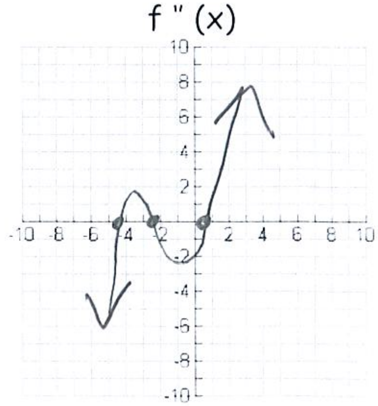
F ' (x)



f (x)



f " (x)



g. $2\sin x\cos x$

h.

x

j. $2+\frac{2}{x^3}$

k.

cy

m. $y'=\frac{2x+3x^2}{3y^2}$

n.

p. $\frac{12x^3}{x^4-1}$

q.

s. $9x^2e^{3x^3}$

t.

v. $y'=\frac{-2ye^{2xy}-2x}{2xe^{2xy}-1}$